Higher Mathematics

Straight Lines - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 5, (2)

$5x + 3y - 6 = 0$

Rearranging gives

$$y = -\frac{5}{3}x + 2$$

Gradient of this line is $-\frac{5}{3}$

Since L is parallel it also has gradient $-\frac{5}{3}$

Using $y - b = m(x - a)$ with (-2, -1) gives

$$y - (-1) = -\frac{5}{3}(x - (-2))$$

Rearranging gives

$$y = -\frac{5}{3}x - \frac{8}{3}$$

2013 Paper 2 Question 2, (3) (3) (3)

a) $m_{PQ} = \frac{6-2}{5-7} = -2$

$m_{QR} = \frac{1}{2}$ since PQ & QR are perpendicular.

Using $y - b = m(x - a)$ with $Q = (5, 6)$ we have

$y - 6 = \frac{1}{2}(x - 5)$

$y = \frac{1}{2}x + \frac{7}{2}$

b) $x + 3y = 13$

$y = -\frac{1}{3}x + \frac{13}{3}$

Equating QR with PT gives

$\frac{1}{2}x + \frac{7}{2} = -\frac{1}{3}x + \frac{13}{3}$

Multiply through by 6 to simplify

$3x + 21 = -2x + 26$

$x = 1$

Substituting $x = 1$ into $y = \frac{1}{2}x + \frac{7}{2}$ gives

$y = \frac{1}{2} + \frac{7}{2} = 4$

$T = (1, 4)$

c) $\overrightarrow{QT} = \binom{-4}{2}$

So, $R = (1 - 4, 4 - 2) = (-3, 2)$

$\overrightarrow{QP} = \binom{2}{-4}$

So, $S = (-3 + 2, 2 - 4) = (-1, -2)$

2014 Paper 1 Question 2, (2)

$$m_{CT} = \frac{2 - (-1)}{1 - 3} = -\frac{3}{2}$$

$m_{tan} = \frac{2}{3}$ since perpendicular

Using $y - b = m(x - a)$ with $(3, -1)$ gives

$$y - (-1) = \frac{2}{3}(x - 3)$$

$$y = \frac{2}{3}x - 3$$

2014 Paper 2 Question 1, (4) (2) (2)

a) $m_{AB} = \frac{2-0}{5-3} = 1$

So, gradient of perpendicular line is -1.

Midpoint of AB $= (4, 1)$.

Using $y - b = m(x - a)$ with $(4, 1)$ we have

$y - 1 = -1(x - 4)$

$y = -x + 5$ (1)

b) $y + 2x = 6$

$y = -2x + 6 \qquad\qquad (2)$

Equating (1) & (2) gives

$-x + 5 = -2x + 6$

$x = 1$

When $x = 1, y = 4$

So, $T = (1, 4)$

c) $A = (3, 0), T = (1, 4)$

$m_{AT} = \frac{4-0}{1-3} = -2$

Using $y - b = m(x - a)$ with $(1, 4)$ gives

$y - 4 = -2(x - 1)$

Rearranging gives

$y = -2x - 2$

2015 Paper 1 Question 9, (3)

$y + \sqrt{3}x = 0$

$y = -\sqrt{3}x$

So, AB has a gradient of $-\sqrt{3}$ since parallel.

$\tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}$

So, BC has a gradient of $-\frac{\sqrt{3}}{3}$

Thus A,B, C cannot be collinear since the gradients are not the same.

2015 Paper 2 Question 1, (4) (3)

a) $m_{AB} = \frac{7-(-5)}{-5-(-1)} = -3$

$m_{alt} = \frac{1}{3}$ since perpendicular to AB.

Using $y - b = m(x - a)$ with $(13, 3)$ gives

$y - 3 = \frac{1}{3}(x - 13)$

Rearranging gives

$y = \frac{1}{3}x + \frac{26}{3}$

b) Mid-point of AC $= (4, 5)$ by inspection

$m_{med} = \frac{5-(-5)}{4-(-1)} = 2$

Using $y - b = m(x - a)$ with $(4, 5)$ we have

$y - 5 = 2(x - 4)$

$y = 2x - 3$

c) Equating the two lines gives

$\frac{1}{3}x + \frac{26}{3} = 2x - 3$

$x + 26 = 6x - 9$

$5x = 35$

$x = 7$

When $x = 7, y = 11$

So, the point of intersection is $(7, 11)$.

2016 Paper 1 Question 1, (2)

$y + 4x = 7$

$y = -4x + 7$

Any line parallel to this has gradient of -4.

Using $y - b = m(x - a)$ with $(-2, 3)$ gives

$y - 3 = -4(x - (-2)$

$y - 3 = -4(x + 2)$

$y = -4x - 5$

2016 Paper 2 Question 1, (1) (2) (3)

a) i) $M = (2, 4)$

ii) $m_{PM} = \frac{-4-4}{0-2} = 4$

Using $y - b = m(x - a)$ with $(2, 4)$ gives

$y - 4 = 4(x - 2)$

$y = 4x - 4$

b) $m_{PR} = \frac{6 - (-4)}{10 - 0} = 1$

So, $m_{perp} = -1$

Using $y - b = m(x - a)$ with $(2, 4)$ gives

$y - 4 = -1(x - 2)$

$y = -x + 6$

c) Midpoint of PR $= (5, 1)$

Test whether $(5, 1)$ lies on L

Substituting $x = 5$ into L gives

$y = -5 + 6 = 1$

So, $(5, 1)$ does lie on the line L.

2017 Paper 1 Question 7, (3)

Midpoint of AB $= (2, 7)$

Let M $= (2, 7)$

$m_{CM} = \frac{11 - 7}{2 - 2} =$ undefined

So, the line is vertical and since it goes through $(2, 7)$ has equation $x = 2$

2017 Paper 1 Question 11, (3)

$3y - 2x = 4$

$$y = \frac{2}{3}x + \frac{4}{3}$$

The gradient of this line is $\frac{2}{3}$.

The gradient of the line joining A&B is also $\frac{2}{3}$ since parallel.

$$m_{AB} = \frac{2 - a}{-7 - 5} = \frac{2}{3}$$

$$\frac{2-a}{-12}=\frac{2}{3}$$

$2-a=-8$

$a=10$

2017 Paper 2 Question 1, (4) (2) (2)

a) $m_{BC}=\frac{0-(-2)}{3-9}=-\frac{1}{3}$

$m_{perp}=3$

Midpoint of BC $=(6,-1)$.

Using $y-b=m(x-a)$ with $(6,-1)$ gives

$y-(-1)=3(x-1)$

$y=3x-19$

b) $m_{AB}=\tan 45°=1$

Using $y-b=m(x-a)$ with $(3,0)$ gives

$y-0=(x-3)$

$y=x-3$

c) Equating gives $3x-19=x-3$

$2x=16$

$x=8$

When $x=8, y=5$

So, the point of intersection is (8, 50).

2018 Paper 1 Question 1, (3)

Mid-point of PQ $=(1,2)$

Let $s=(1,2)$

$$m_{RS}=\frac{6-2}{3-1}=2$$

Using $y-b=m(x-a)$ with $(1,2)$ gives

$y-2=2(x-1)$

$y - 2 = 2x - 2$

$y = 2x$

2018 Paper 1 Question 5, (1) (1)

a) $\frac{8}{10} = \frac{4}{5}$

b) $\frac{4}{5}$ of $(9 - 4)$

$= \frac{4}{5}\ x\ 5$

$= 4$

$t = 4$

2018 Paper 1 Question 8, (2)

$y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

Gradient $= \sqrt{3}$

So, $\tan\theta = \sqrt{3}$

$\theta = 60°$ from exact values.

2018 Paper 2 Question 5, (3) (2)

a) Mid-point $PQ = (6, 1)$

$m_{PQ} = \frac{4 + 2}{3 - 9} = \frac{6}{-6} = -1$

So, $m_{L1} = 1$

$y - b = m(x - a)$

$y - 1 = x - 6$

$y = x - 5$

b) $3y + x = 25$

$3y = 25 - x$

$y = x - 5$

$3y = 3x - 15$

Equating gives

$3x - 15 = 25 - x$

$4x = 40$

$x = 10$

When $x = 10, y = 10 - 5 = 5$

So, $C = (10, 5)$

2019 Paper 1 Question 7, (4)

$\tan 30° = m = \frac{1}{\sqrt{3}}$

Using $y - b = m(x - a)$ gives

$y + 4 = \frac{1}{\sqrt{3}}(x - 0)$

$y + 4 = \frac{1}{\sqrt{3}}x$

$y = \frac{1}{\sqrt{3}}x - 4$

2019 Paper 2 Question 1, (3) (3) (2)

a) Since D is the mid-point of AC it has co-ordinates $(-4, -3)$.

$m_{BD} = \frac{-3 + 8}{-4 - 11} = -\frac{1}{3}$

Using $y - b = m(x - a)$ with $(11, -8)$ gives

$y + 8 = -\frac{1}{3}(x - 11)$

$3y + 24 = -x + 11$

$3y = -x - 13 \qquad (1)$

b) $m_{BC} = \frac{6+8}{-3-11} = \frac{14}{-14} = -1$

$m_{AE} = 1$ since AE & BC are perpendicular $\rightarrow m_{BC} \cdot m_{AE} = -1$

Using $y - b = m(x - a)$ with $(-5, -12)$ gives

$y + 12 = x + 5$

$y = x - 7$ (2)

c) By substituting (2) into (1) we have

$3(x - 7) = -x - 13$

$3x - 21 = -x - 13$

$4x = 8$

$x = 2$

When $x = 2, y = 2 - 7 = -5$ using equation (2)

So, the point of intersection is $(2, -5)$.

2022 Paper 1 Question 1, (3)

$5x + 2y = 7$

$2y = -5x + 7$

$y = \frac{-5}{2}x + \frac{7}{2}$

So, the gradient is $\frac{-5}{2}$.

The perpendicular gradient is $\frac{2}{5}$.

Using $y - b = m(x - a)$ with $(-1, 6)$ gives

$y - 6 = \frac{2}{5}(x - (-1))$

$y - 6 = \frac{2}{5}(x + 1)$

$5y - 30 = 2x + 2$

$5y = 2x + 32$

$y = \frac{2}{5}x + \frac{32}{5}$

2022 Paper 1 Question 5, (3)

Angle in positive direction of x-axis is $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Gradient $= tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Using $y - b = m(x - a)$ with $(-2, 0)$ gives

$y - 0 = \dfrac{1}{\sqrt{3}}(x - (-2))$

$y - 0 = \dfrac{1}{\sqrt{3}}(x + 2)$

$y = \dfrac{1}{\sqrt{3}}x + \dfrac{2}{\sqrt{3}}$

2022 Paper 2 Question 1, (3) (3) (2)

a) $m_{AB} = \frac{-4-(-1)}{2-(-1)} = -\frac{3}{3} = -1$

$m_{perp} = 1$

Using $y - b = m(x - a)$ with $(7, 3)$ gives

$y - 3 = 1(x - 7)$

$y - 3 = x - 7$

$y = x - 4$

b) The midpoint of AC is $(3, 1)$.

Using points $(2, -4)$ and $(3, 1)$ gives

$m = \frac{-4-1}{2-3} = \frac{-5}{-1} = 5$

Using $y - b = m(x - a)$ with $(3, 1)$ gives

$y - 1 = 5(x - 3)$

$y - 1 = 5x - 15$

$y = 5x - 14$

c) Equate the lines from parts **a)** and **b)** to give

$x - 4 = 5x - 14$

$-4x = -10$

$x = 2.5$

When $x = 2.5, y = 2.5 - 4 = -1.5$

So, the point of intersection is $(2.5, -1.5)$

2023 Paper 1 Question 2, (4)

$$m_{PQ} = \frac{0-6}{10-(-2)} = \frac{-6}{12} = -\frac{1}{2}$$

$m_{perp} = 2$

Midpoint of PQ $= (4, 3)$

Using $y - b = m(x - a)$ with $(4, 3)$ gives

$y - 3 = 2(x - 4)$

$y - 3 = 2x - 8$

$y = 2x - 5$

2023 Paper 2 Question 1, (3) (2)

a) $m_{QR} = \frac{8-3}{-2-13} = \frac{5}{-15} = -\frac{1}{3}$

$m_{perp} = 3$

Using $y - b = m(x - a)$ with $(5, -1)$ gives

$y - (-1) = 3(x - 5)$

$y + 1 = 3x - 15$

$y = 3x - 16$

b) $m_{PR} = \frac{3-(-1)}{13-5} = \frac{4}{8} = \frac{1}{2}$

$tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

So, angle $= 26.6°$

2024 Paper 1 Question 1, (3)

$m = \tan 30° = \frac{1}{\sqrt{3}}$

The y-intercept is at 4 giving

$y = \frac{1}{\sqrt{3}}x + 4$

2024 Paper 2 Question 1, (3) (3) (2)

a) Midpoint of AC $= (4, 4)$. Let this point be M.

$M_{BM} = \frac{4-(-6)}{4-(-1)} = 2$

Using $y - b = m(x - a)$ with $(4, 4)$ gives

$y - 4 = 2(x - 4)$

$y - 4 = 2x - 8$

$y = 2x - 4$

b) $M_{BC} = \frac{0-(-6)}{11-(-1)} = \frac{1}{2}$

$M_{perp} = -2$

Using $y - b = m(x - a)$ with $(11, 0)$ gives

$y - 0 = -2(x - 11)$

$y = -2x + 22$

c) Equate the line equations to give

$2x - 4 = -2x + 22$

$4x = 26$

$x = \frac{13}{2}$

Substitute $x = \frac{13}{2}$ into $y = 2x - 4$ to give

$y = 2(\frac{13}{2}) - 4$

$\quad = 9$

So, the point of intersection is $(\frac{13}{2}, 9)$

2025 Paper 1 Question 2, (4)

Midpoint of $AB = (5, 7)$

$$M_{AB} = \frac{10-4}{9-1} = \frac{6}{8} = \frac{3}{4}$$

$$M_{perp} = -\frac{4}{3}$$

Using $y - b = m(x - a)$ with $(5, 7)$ gives

$$y - 7 = -\frac{4}{3}(x - 5)$$

$$3y - 21 = -4x + 20$$

$$3y = -4x + 41$$

$$y = -\frac{4}{3}x + \frac{41}{3}$$

2025 Paper 2 Question 1, (3) (3) (2)

a) $m_{AC} = \frac{-24-(-14)}{21-(-9)} = \frac{-10}{30} = -\frac{1}{3}$

$m_{perp} = 3$

Using $y - b = m(x - a)$ with $(9, 20)$ gives

$$y - 20 = 3(x - 9)$$

$$y - 20 = 3x - 27$$

$$y = 3x - 7$$

b) Midpoint of BC $= (15, -2)$

$m = \frac{-14-(-2)}{-9-15} = \frac{-12}{-24} = \frac{1}{2}$

Using $y - b = m(x - a)$ with $(15, -2)$ gives

$$y - (-2) = \tfrac{1}{2}(x - 15)$$

$$y + 2 = \tfrac{1}{2}(x - 15)$$

$$2y + 4 = x - 15$$

$$2y = x - 19$$

$$y = \tfrac{1}{2}x - \tfrac{19}{2}$$

c) Equate the lines from **a)** and **b)** to give

$3x - 7 = \frac{1}{2}x - \frac{19}{2}$

$6x - 14 = x - 19$

$5x = -5$

$x = -1$

Substitute $x = 1$ into $y = 3x - 7$ to give $y = 3(-1) - 7 = -10$

So, the point of intersection is $(-1, -10)$

2026 Paper 2 Question 8, (4) (2)

a) Midpoint of AB = $(\frac{-2+10}{2}, \frac{11+(-13)}{2}) = (4, -1)$

$m_{AB} = \frac{-13-11}{10-(-2)} = \frac{-24}{12} = -2$ giving $m_{perp} = \frac{1}{2}$

Using $y - b = m(x - a)$ gives

$y - (-1) = \frac{1}{2}(x - 4)$

$y + 1 = \frac{1}{2}x - 2$

$y = \frac{1}{2}x - 3$

b) $y + x = 9$ can be rearranged to give $y = -x + 9$

This line has a gradient of -1 so the angle it makes with the positive direction of the x axis is $135°$ (since $tan^{-1}(1) = 45°$ and using CAST).

The line $y = \frac{1}{2}x - 3$ has a gradient of $\frac{1}{2}$

$m = \tan\theta$

$\tan\theta = \frac{1}{2}$

$\theta = tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

$135 = 26.6 = 108.8°$