Higher Mathematics

Rates of Change - Solutions

Marks are indicated in brackets after each question number

2014 Paper 2 Question 9, (3) (2) (3)

a) $v(t) = 8\cos(2t - \frac{\pi}{2})$

$a(t) = v'(t) = -16\sin(2t - \frac{\pi}{2})$

b) $v'(10) = -16\sin\left(20 - \frac{\pi}{2}\right) = 6.53$

Since $6.53 > 0$ the velocity of the object is increasing when $t = 10$.

c) $s(t) = \int v(t)\, dt$

$= \int 8\cos(2t - \frac{\pi}{2})\, dt$

$= 4\sin\left(2t - \frac{\pi}{2}\right) + c$

Since $s(t) = 4$ when $t = 0$ we have

$4 = 4\sin(0 - \frac{\pi}{2}) + c$

$4 = 4\sin\left(-\frac{\pi}{2}\right) + c$

$4 = 4 \cdot (-1) + c$

$c = 8$

So, $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$.

2015 Paper 1 Question 15, (6)

$\frac{dT}{dt} = \frac{1}{25}t - k$

$T(t) = \int \frac{dT}{dt}\, dt$

$= \int (\frac{1}{25}t - k)\, dt$

$= \frac{1}{50}t^2 - kt + c$

Now we must find k & c using the information given in the question.

At $t = 0, T = 100$ (note that 'initially' means $t = 0$).

So, $100 = \left(\frac{1}{50} \cdot 0\right) - (k \cdot 0) + c$

$c = 100$

At $t = 10, T = 82$.

So, $82 = \left(\frac{1}{50} \cdot 10^2\right) - 10k + 100$

$82 = 2 - 10k + 100$

$10k = 20$

$k = 2$

So, $T(t) = \frac{1}{50}t^2 - 2t + 100$.

2017 Paper 1 Question 8, (3)

$$d(t) = \frac{1}{2t} = \frac{t^{-1}}{2} = \frac{1}{2}t^{-1}$$

$$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$$

$$d'(5) = -\frac{1}{2 \cdot 5^2} = -\frac{1}{50}$$

So, the rate of change of $d(t)$ when $t = 5$ is $-\frac{1}{50}$.

2023 Paper 2 Question 5, (3)

$f(x) = (3 - 2x)^4$

$f'(x) = 4(3 - 2x)^3 \cdot -2$

$= -8(3 - 2x)^3$

$f'(4) = -8(3 - 2(4))^3$

$= -8(-5)^3$

$= 1000$

So, the rate of change is 1000.