



Higher Mathematics

Polynomials - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 6, (2)

$$\text{Let } f(x) = x^3 + 3x^2 - 5x - 6$$

$$f(2) = 2^3 + 3 \cdot 2^2 - 5 \cdot 2 - 6 = 4$$

The remainder is 4 when $f(x)$ is divided by $(x - 2)$.

2013 Paper 1 Question 17, (2)

$$y = kx(x + a)^2$$

Rewriting this gives

$$y = k(x - 0)(x + a)(x + a)$$

This shows that the roots are at $x = 0, x = -a$

So, by inspection, $a = 2$.

$$\text{So, } y = kx(x + 2)^2$$

Substituting the point (1, 3) gives

$$3 = k(1 + 2)^2$$

$$3 = 9k$$

$$k = \frac{1}{3}$$

2013 Paper 2 Question 3, (4) (5)

a) Let $f(x) = x^3 + 3x^2 + x - 5$

Using synthetic division gives

1	1	3	1	-5
		1	4	5
	1	4	5	0



$$f(x) = (x - 1)(x^2 + 4x + 5)$$

Note that $x^2 + 4x + 5$ does not factorise.

b) Let $g(x) = x^4 + 4x^3 + 2x^2 - 20x + 3$

$$g'(x) = 4x^3 + 12x^2 + 4x - 20$$

Stationary Points occur where $g'(x) = 0$ giving

$$4x^3 + 12x^2 + 4x - 20 = 0$$

$$x^3 + 3x^2 + x - 5 = 0$$

$$(x - 1)(x^2 + 4x + 5) = 0 \text{ from a)}$$

The solution to this equation is $x = 1$ since $x^2 + 4x + 5$ has no solutions

$$(b^2 - 4ac = -4 < 0)$$

Thus $g(x)$ has only one stationary point.

2014 Paper 1 Question 15, (2)

Since the roots are at $x = -1, x = 2$ we have $y = k(x + 1)(x - 2)^2$ for some value k

Substituting the point $(0, -8)$ gives

$$-8 = k(0 + 1)(0 - 2)^2$$

$$-8 = 4k$$

$$k = -2$$

$$\text{So, } y = -2(x + 1)(x - 2)^2.$$

2014 Paper 1 Question 22, (4) (3)

a) Let $f(x) = 6x^3 + 7x^2 + ax + b$

Since $x + 1$ is a factor $f(-1) = 0$

$$f(-1) = -6 + 7 - a + b = 0$$

$$\text{Rearranging gives } a = b + 1 \quad (1)$$

Since 72 is the remainder when $f(x)$ is divided by $x - 2$ we have

$$f(2) = 48 + 28 + 2a + b = 72$$

$$\text{Rearranging gives } 2a = -b - 4 \quad (2)$$



Substituting (1) into (2) gives

$$2(b + 1) = -b - 4$$

$$2b + 2 = -b - 4$$

$$3b = -6$$

$$b = -2$$

Using (1) we have

$$a = -2 + 1 = -1$$

b) $f(x) = 6x^3 + 7x^2 - x - 2$

Since $x + 1$ is a factor we have

-1	6	7	-1	-2
		-6	-1	2
	6	1	-2	0

$$\begin{aligned}\text{So, } f(x) &= (x + 1)(6x^2 + x - 2) \\ &= (x + 1)(3x + 2)(2x - 1)\end{aligned}$$

2015 Paper 1 Question 3, (4)

Let $f(x) = x^3 - 3x^2 - 10x + 24$

Using synthetic division gives

-3	1	-3	-10	24
		-3	18	-24
	1	-6	8	0

Since the remainder is zero, $(x + 3)$ is a factor of $f(x)$

$$\begin{aligned}f(x) &= (x + 3)(x^2 - 6x + 8) \\ &= (x + 3)(x - 4)(x - 2)\end{aligned}$$



2016 Paper 1 Question 15, (3) (1)

a) $f(x) = k(x - a)(x - b)^2$

By inspection of the roots we have

$$f(x) = k(x - 4)(x + 5)^2$$

Substituting the point (1, 9) gives

$$9 = k(1 - 4)(1 + 5)^2$$

Rearranging and solving for k gives

$$k = -\frac{1}{12}$$

b) $g(x) = f(x) - d$

Since d is positive the graph of $g(x)$ is the graph of $f(x)$ moved down by ' d ' units

By considering the graph of $f(x)$ it must be moved down by at least 9 units to give only one root – in other words to move the point (1, 9) below the x axis. So, $d > 9$

2016 Paper 2 Question 3, (2) (3) (1) (4)

a) i) Let $f(x) = 2x^3 - 9x^2 + 3x + 14$

Using synthetic division gives

-1		2	-9	3	14
			-2	11	-14
		2	-11	14	0

Since the remainder is zero $(x + 1)$ is a factor of $f(x)$

ii) $2x^3 - 9x^2 + 3x + 14 = 0$

$$(x + 1)(2x^2 - 11x + 14) = 0$$

$$(x + 1)(2x - 7)(x - 2) = 0$$

$$x = -1, x = 2, x = \frac{7}{2}$$

b) i) $y = 2x^3 - 9x^2 + 3x + 14$

$$y = (x + 1)(2x - 7)(x - 2)$$

By inspection of the roots $A = (-1, 0), B = (2, 0)$.



$$\begin{aligned}\text{ii) Area} &= \int_{-1}^2 (2x^3 - 9x^2 + 3x + 14) dx \\&= \left[\frac{2}{4}x^4 - \frac{9}{3}x^3 + \frac{3}{2}x^2 + 14x \right]_{-1}^2 \\&= \left[\frac{1}{2}x^4 - 3x^3 + \frac{3}{2}x^2 + 14x \right]_{-1}^2 \\&= (8 - 24 + 6 + 28) - \left(\frac{1}{2} + 3 + \frac{3}{2} - 14 \right) \\&= 27 \\ \text{Area} &= 27 \text{ units}^2\end{aligned}$$

2017 Paper 2 Question 2, (2) (3)

a) $f(x) = 2x^3 - 5x^2 + x + 2$

Using synthetic division gives

1		2	-5	1	2
			2	-3	-2
		2	-3	-2	0

Since the remainder is zero, $x - 1$ is a factor of $f(x)$.

$$\begin{aligned}\text{b) } f(x) &= 2x^3 - 5x^2 + x + 2 \\&= (x - 1)(2x^2 - 3x - 2) \\&= (x - 1)(2x + 1)(x - 2)\end{aligned}$$

For $f(x) = 0$ we have

$$(x - 1)(2x + 1)(x - 2) = 0$$

$$x = -\frac{1}{2}, x = 1, x = 2$$

2018 Paper 1 Question 7, (1) (3) (4)

a) (0, 5)

b) $y = x^3 - 3x^2 + 2x + 5$

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$



$$m_{tan} = 3(0)^2 - 6(0) + 2 = 2$$

$$y = 2x + 5$$

c) $2x + 5 = x^3 - 3x^2 + 2x + 5$

$$x^3 - 3x^2 = 0$$

$$x^2(x - 3) = 0$$

$$x = 0, x = 3$$

$$\text{When } x = 3, y = (2 \times 3) + 5 = 11$$

$$Q = (3, 11)$$

2018 Paper 1 Question 15, (5)

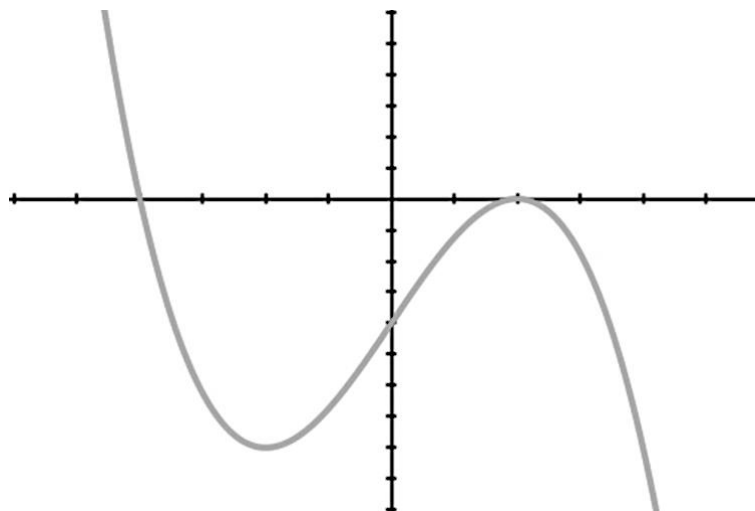
Since $(x + 4)$ is a factor, $x = -4$ is a root.

Since $x = 2$ is a repeated root, $(x - 2)$ is a repeated factor.

So, the equation of the function can be written $f(x) = k(x + 4)(x - 2)(x - 2)$.

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$.

Considering all of the above information and assuming a k value (which cannot be determined from the information) a possible graph is:





2018 Paper 2 Question 7, (2) (2)

a) i)

2	2	-3	-3	2
		4	2	-2
	2	1	-1	0

Zero remainder shows that $(x - 2)$ is a factor.

$$\begin{aligned}\text{ii) } 2x^3 - 3x^2 - 3x + 2 &= (x - 2)(2x^2 + x - 1) \\ &= (x - 2)(2x - 1)(x + 1)\end{aligned}$$

2019 Paper 2 Question 10, (2) (5)

a)

- 3	3	10	1	- 8	- 6
		- 9	- 3	6	6
	3	1	- 2	- 2	0

Since the remainder is 0, $(x + 3)$ is a factor.

$$\begin{aligned}\text{b) } 3x^4 + 10x^3 + x^2 - 8x - 6 \\ = (x + 3)(3x^3 + x^2 - 2x - 2)\end{aligned}$$

We need to factorise $3x^3 + x^2 - 2x - 2$.

Let $g(x) = 3x^3 + x^2 - 2x - 2$.

By inspection, $(x - 1)$ is a factor since $g(1) = 0$.



1	3	1	-2	-2
		3	4	2
	3	4	2	0

So, $3x^4 + 10x^3 + x^2 - 8x - 6 = (x + 3)(x - 1)(3x^2 + 4x + 2)$

Note this does not factorise any further.

2022 Paper 1 Question 13, (2) (3) (1)

a) i)

-2	1	-2	-20	-24
		-2	8	24
	1	-4	-12	0

Since the remainder is 0, $(x + 2)$ is a factor of $f(x)$.

ii) $f(x) = 0$

$$x^3 - 2x^2 - 20x - 24 = 0$$

Using the synthetic division table quotient gives

$$(x + 2)(x^2 - 4x - 12) = 0$$

$$(x + 2)(x - 6)(x + 2) = 0$$

$$x = -2, x = 6$$

b) $k = 3$ since the stationary point on the graph of $y = f(x)$ is at $(-2, 0)$ and so has been moved by '3' units to the right.



2023 Paper 1 Question 10, (2) (5)

a)

-5	1	3	-7	9	-30
		-5	10	-15	30
	1	-2	3	-6	0

Since the remainder is zero, $(x + 5)$ is a factor.

b) $x^4 + 3x^3 - 7x^2 + 9x - 30 = 0$

$$(x + 5)(x^3 - 2x^2 + 3x - 6) = 0$$

To fully factorise we have to factorise $x^3 - 2x^2 + 3x - 6$ using synthetic division.

Since we have no factor we have to use trial and error with factors of -6 .

2	1	-2	3	-6
		2	0	6
	1	0	3	0

Since the remainder is zero, $(x - 2)$ is a factor. We can now fully factorise.

$$(x + 5)(x - 2)(x^2 + 3) = 0$$

$$x + 5 = 0$$

$$x = -5$$

$$x - 2 = 0$$

$$x = 2$$

$$x^2 + 3 = 0$$

No solutions

2024 Paper 1 Question 10, (2) (4)

a)

1	2	3	-4	-3	2
		2	5	1	-2
	2	5	1	-2	0



Since the remainder is zero $(x - 1)$ is a factor.

b) $2x^4 + 3x^3 - 4x^2 - 3x + 2$

$$= (x - 1)(2x^3 + 5x^2 + x - 2)$$

We have to use synthetic division to factorise $2x^3 + 5x^2 + x - 2$

-1	2	5	1	-2
		-2	-3	2
	2	3	-2	0

$$\text{So, } 2x^3 + 5x^2 + x - 2 = (x + 1)(2x^2 + 3x - 2)$$

Note that $2x^2 + 3x - 2$ does not factorise

$$\begin{aligned}\text{So, } 2x^4 + 3x^3 - 4x^2 - 3x + 2 &= (x - 1)(2x^3 + 5x^2 + x - 2) \\ &= (x - 1)(x + 1)(2x^2 + 3x - 2)\end{aligned}$$

2024 Paper 2 Question 13, (3)

From the root points $x = -1, x = 3$ and $x = 5$ we have

$$f(x) = k(x - 3)^2(x + 1)(x - 5)$$

Substitute the point $(0, -9)$ to give

$$-9 = k(0 - 3)^2(0 + 1)(0 - 5)$$

$$-9 = k(9)(1)(-5)$$

$$-9 = -45k$$

$$k = \frac{9}{45} = \frac{1}{5}$$

$$\text{So, } f(x) = \frac{1}{5}(x - 3)^2(x + 1)(x - 5)$$



2025 Paper 1 Question 7, (2) (3)

a)

-3	5	16	-1	-12
		-15	-3	12
	5	1	-4	0

Since the remainder is zero $(x + 3)$ is a factor.

b) $5x^3 + 16x^2 - x - 12 = 0$

$$(x + 3)(5x^2 + x - 4) = 0$$

$$(x + 3)(5x - 4)(x + 1) = 0$$

$$x = -3, x = \frac{4}{5}, x = -1$$

2026 Paper 1 Question 11, (2) (2) (4)

a) i)

-2	1	7	18	16
		-2	-10	-16
	1	5	8	0

Since the remainder is zero $(x + 2)$ is a factor.

a) ii) $x^3 + 7x^2 + 18x + 16$

$$= (x + 2)(x^2 + 5x + 8)$$

Since $x^2 + 5x + 8$ does not factorise $(x + 2)$ is the only linear factor.

b) Equate the two curves to give

$$2x^3 + 20x^2 + 27x + 9 = 6x^2 - 9x - 23$$



$$2x^3 + 14x^2 + 36x + 32 = 0$$

$$2(x^3 + 7x^2 + 18x + 16) = 0$$

$$2(x + 2)(x^2 + 5x + 8) = 0$$

$$x + 2 = 0$$

$$x = -2$$

Substitute $x = -2$ into $6x^2 - 9x - 23$ to give

$$6(-2)^2 - 9(-2) - 23$$

$$= 19$$

Giving the point $(-2, 19)$

$$x^2 + 5x + 8 = 0$$

$$a = 1, b = 5, c = 8$$

$$b^2 - 4ac = 5^2 - 4(1)(8)$$

$$= 25 - 32$$

$$= -7$$

Since $b^2 - 4ac < 0$ $x^2 + 5x + 8 = 0$ has no solutions.

So, the only point of intersection is $(-2, 19)$