



Higher Mathematics

Optimization - Questions

Marks are indicated in brackets after each question number

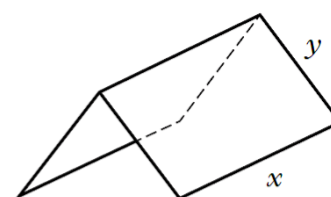
2013 Paper 2 Question 7, (3) (7)

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2 .

(a) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

(b) These rods cost £8.25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

- Find the value of x for which L is a minimum.
- Calculate the minimum cost of a frame.

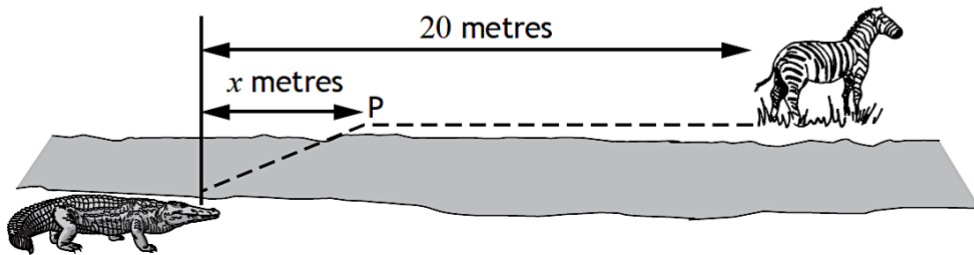


2015 Paper 2 Question 8, (1) (1) (8)

A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T , measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

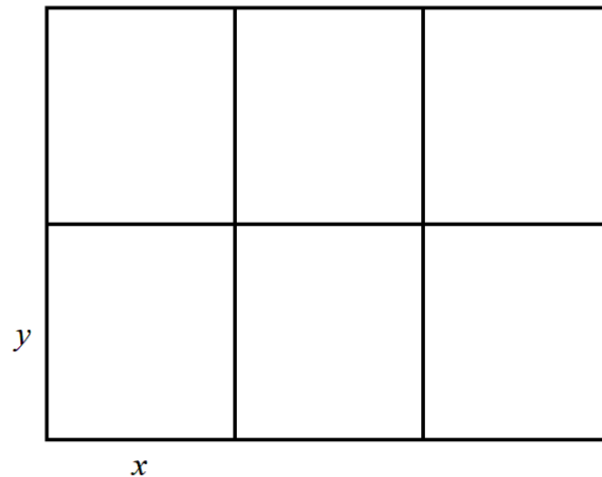
- (a) (i) Calculate the time taken if the crocodile does not travel on land.
- (ii) Calculate the time taken if the crocodile swims the shortest distance possible.
- (b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time.



2016 Paper 2 Question 7, (3) (6)

A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



- (a) The area of land being set aside is 108 m^2 .

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$

- (b) Find the value of x that minimises the length of fencing required.

2018 Paper 2 Question 9, (6)

A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

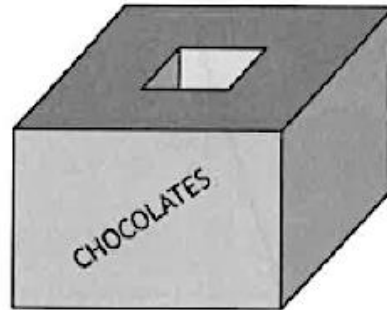
$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P .



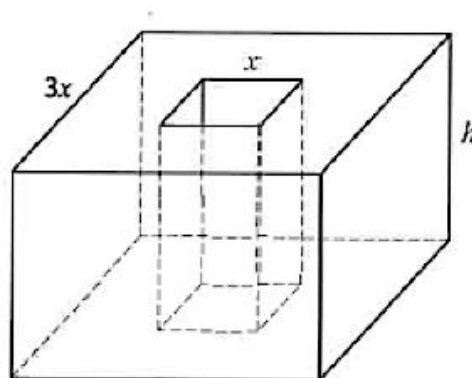
2019 Paper 2 Question 11, (3) (6)

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is 2000 cm^3



- (a) Show that the total surface area, $A \text{ cm}^2$, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$

- (b) To minimise the cost of production, the surface area, A , of the box should be as small as possible.

Find the minimum value of A .

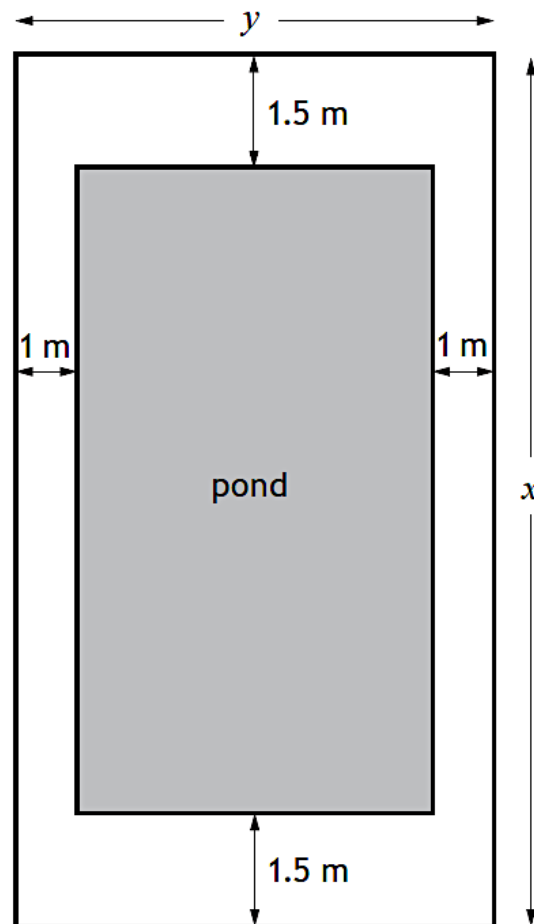


2022 Paper 2 Question 8, (3) (6)

A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

$$A(x) = 156 - 2x - \frac{450}{x}.$$

(b) Determine the maximum area of the pond.

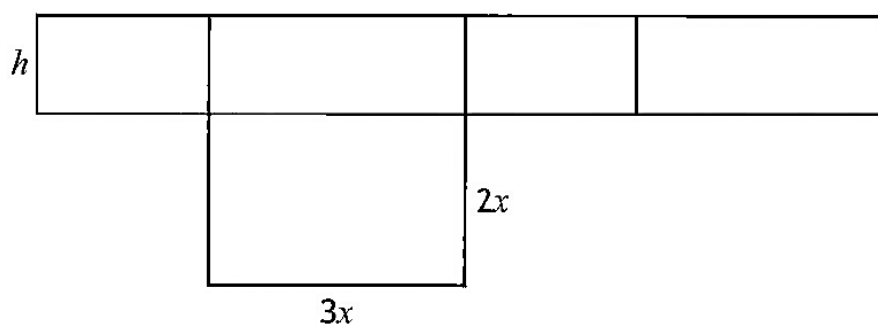


2023 Paper 2 Question 14, (1) (2) (4)

A net of an open box is shown.

The box is a cuboid with height h centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.



- (a) (i) Express the area of the net, $A \text{ cm}^2$, in terms of h and x .
- (ii) Given that $A = 7200 \text{ cm}^2$, show that the volume of the box, $V \text{ cm}^3$, is given by $V = 4320x - \frac{18}{5}x^3$.
- (b) Determine the value of x that maximises the volume of the box.



2025 Paper 2 Question 10, (3) (6)

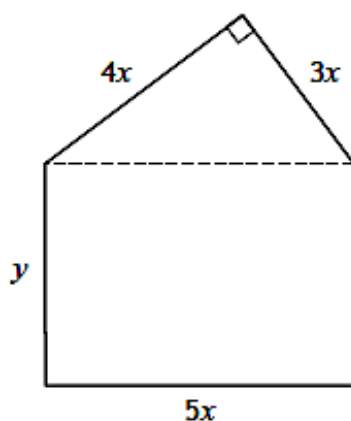
A hotel owner is designing signs showing the room numbers.



Each sign is a rectangle with a right-angled triangle above it.

The length and breadth of the rectangle are $5x$ centimetres and y centimetres respectively.

The shorter sides of the triangle are $3x$ centimetres and $4x$ centimetres.



The area of the sign is 150 square centimetres.

(a) Show that the perimeter, P cm, of the sign is given by

$$P = 9.6x + \frac{60}{x}.$$

Each sign will be lit using a lighting strip placed around its perimeter.

The hotel owner requires the perimeter, P , of the sign to be as small as possible.

(b) Find the minimum value of P .