Higher Mathematics

Integration - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 7, (2)

$$\int x(3x+2)\,dx$$

$$= \int (3x^2+2x)\,dx$$

$$= x^3 + x^2 + c$$

2013 Paper 1 Question 16, (2)

$$\int (1-6x)^{-\frac{1}{2}}\,dx$$

$$= \frac{(1-6x)^{\frac{1}{2}}}{\frac{1}{2}\cdot(-6)} + c$$

$$= -\frac{1}{3}(1-6x)^{\frac{1}{2}} + c$$

2013 Paper 2 Question 6, (5)

$$\int_0^a 5\sin 3x\,dx = \frac{10}{3}$$

$$\left[-\frac{5}{3}\cos 3x\right]_0^a = \frac{10}{3}$$

$$(-\tfrac{5}{3}\cos 3a) - (-\tfrac{5}{3}\cos 0) = \tfrac{10}{3}$$

$$-\frac{5}{3}\cos 3a + \frac{5}{3} = \frac{10}{3}$$

$$-\frac{5}{3}\cos 3a = \frac{5}{3}$$

$$\cos 3a = -1$$

$$3a = \pi, a = \frac{\pi}{3}$$

There are no other solutions in $0 \leq a < \pi$

2014 Paper 1 Question 5, (2)

$$\int (2x+9)^5 \, dx$$

$$= \frac{(2x+9)^6}{6 \cdot 2} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

2014 Paper 2 Question 5, (5)

$$\int_4^t (3x+4)^{-\frac{1}{2}} \, dx = 2$$

Using the reverse chain rule we have

$$\left[\frac{2}{3}(3x+4)^{\frac{1}{2}}\right]_4^t = 2$$

$$\frac{2}{3}(3t+4)^{\frac{1}{2}} - \frac{2}{3}(12+4)^{\frac{1}{2}} = 2$$

$$\frac{2}{3}(3t+4)^{\frac{1}{2}} - \frac{8}{3} = 2$$

Simplifying gives

$$(3t+4)^{\frac{1}{2}} = 7$$

$$3t+4 = 49$$

$$t = 15$$

2015 Paper 1 Question 12, (4)

$$\int_0^{\frac{3\pi}{4}} (a\cos bx) \, dx = \int_0^{\frac{\pi}{4}} (a\cos bx) \, dx + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (a\cos bx) \, dx$$

$$= \tfrac{1}{2} - (2 \cdot \tfrac{1}{2})$$

Note that since the part of the graph between $\frac{\pi}{4}$ & $\frac{3\pi}{4}$ is under the x axis it has a negative integral $= -\frac{1}{2}$.

2015 Paper 2 Question 7, (2) (2) (2)

a) $\int(3\cos 2x+1)\,dx=\frac{3}{2}\sin 2x+x+c$

b) $3\cos 2x+1=3\,(\cos^2 x-\sin^2 x)+1$

$$=3\cos^2 x-3\sin^2 x+1$$

Using $\cos^2 x+\sin^2 x=1$ to give

$$=3\cos^2 x-3\sin^2 x+\cos^2 x+\sin^2 x$$

$$=4\cos^2 x-2\sin^2 x$$

c) $\int(\sin^2 x-2\cos^2 x)\,dx$

$$\sin^2 x-2\cos^2 x=-\frac{1}{2}(4\cos^2 x-2\sin^2 x)$$

$$=-\frac{1}{2}(3\cos 2x+1)$$

$$=-\frac{3}{2}\cos 2x-\frac{1}{2}$$

So, $\int(\sin^2 x-2\cos^2 x)\,dx=\int\left(-\frac{3}{2}\cos 2x-\frac{1}{2}\right)dx$

$$=-\frac{3}{4}\sin 2x-\frac{1}{2}x+c$$

2016 Paper 1 Question 5, (2)

$$\int 8\cos(4x+1)\,dx$$

$$=\frac{8\sin(4x+1)}{4}+c$$

$$=2\sin(4x+1)+c$$

2016 Paper 2 Question 9, (4)

$$f'(x)=\frac{2x+1}{\sqrt{x}}$$

$$f(x)=\int f'(x)\,dx$$

$$=\int\frac{2x+1}{\sqrt{x}}\,dx$$

$$=\int\frac{2x+1}{x^{\frac{1}{2}}}\,dx$$

$$=\int(2x^{\frac{1}{2}}+x^{-\frac{1}{2}})\,dx$$

$= \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

Since $f(9) = 40$ we have

$40 = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

$40 = 36 + 6 + c$

$c = -2$

So, $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2.$

2017 Paper 1 Question 13, (4)

$\int \frac{1}{(5-4x)^{\frac{1}{2}}}\, dx$

$= \int (5-4x)^{-\frac{1}{2}}\, dx$

$= \frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}\cdot(-4)} + c$

$= -\frac{1}{8}(5-4x)^{\frac{1}{2}} + c$

2022 Paper 1 Question 6, (4)

$\int_{-5}^{2} (10-3x)^{-\frac{1}{2}}\, dx$

$= [\frac{(10-3x)^{\frac{1}{2}}}{-\frac{3}{2}}]_{-5}^{2}$

$= [-\frac{2}{3}(10-3x)^{\frac{1}{2}}]_{-5}^{2}$

$= \left[-\frac{2}{3}(10-3(2))^{\frac{1}{2}}\right] - \left[-\frac{2}{3}(10-3(-5))^{\frac{1}{2}}\right]$

$= \left[-\frac{2}{3}\sqrt{4}\right] - \left[-\frac{2}{3}\sqrt{25}\right]$

$= -\frac{4}{3} + \frac{10}{3}$

$= 2$

2022 Paper 2 Question 6, (5)

$$\frac{dy}{dx} = 1 - \frac{3}{x^2} = 1 - 3x^{-2}$$

$$\int 1 - 3x^{-2}\,dx = x + 3x^{-1} + c$$

$$= x + \frac{3}{x} + c$$

So, $y = x + \frac{3}{x} + c$

Substitute $(3, 6)$ to give

$$6 = 3 + \frac{3}{3} + c$$

$$6 = 3 + 1 + c$$

$$c = 2$$

$$y = x + \frac{3}{x} + 2$$

2023 Paper 1 Question 6, (4)

$$\int 2x^5 - 6\sqrt{x}\,dx$$

$$= \int 2x^5 - 6x^{\frac{1}{2}}\,dx$$

$$= \frac{2x^6}{6} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{x^6}{3} - 4x^{\frac{3}{2}} + c$$

2023 Paper 1 Question 11, (3) (1)

a) $\int_{\frac{\pi}{2}}^{\pi} 5\sin x - 3\cos x\ \ dx$

$$= [-5\cos x - 3\sin x]_{\frac{\pi}{2}}^{\pi}$$

$$= [-5\cos\pi - 3\sin\pi] - \left[-5\cos\frac{\pi}{2} - 3\sin\frac{\pi}{2}\right]$$

$$= [5] - [-3]$$

$$= 8$$

b) Shade the area between $\frac{\pi}{2}$ and π.

2023 Paper 2 Question 3, (2)

$$\int 7\cos\left(4x+\frac{\pi}{3}\right)dx$$

$$=\frac{7}{4}\sin\ (4x+\frac{\pi}{3})+c$$

2023 Paper 2 Question 12, (4)

$$\frac{dy}{dx}=8x^3+3$$

$$\int 8x^3+3\ \ dx$$

$$=\frac{8x^4}{4}+3x+c$$

$$=2x^4+3x+c$$

So, $y=2x^4+3x+c$

Substitute $(-1,3)$ to give

$3=2(-1)^4+3(-1)+c$

$3=2-3+c$

$c=4$

So, $y=2x^4+3x+4$

2024 Paper 2 Question 5, (3)

$$\int_0^{\frac{\pi}{7}}\sin 5x\,dx$$

$$=[-\frac{1}{5}\cos 5x]_0^{\frac{\pi}{7}}$$

$$=-\frac{1}{5}\cos\frac{5\pi}{7}-(-\frac{1}{5}\cos 0)$$

$$=-\frac{1}{5}\cos\frac{5\pi}{7}+\frac{1}{5}\cos 0=0.3246$$

2025 Paper 1 Question 3, (4)

$$\int \frac{12}{x^2} + x^{\frac{1}{2}}\, dx$$

$$= \int 12x^{-2} + x^{\frac{1}{2}}\, dx$$

$$= -12x^{-1} + \frac{2}{3}x^{\frac{3}{2}} + c$$

$$= -\frac{12}{x} + \frac{2}{3}x^{\frac{3}{2}} + c$$

2025 Paper 1 Question 12, (4)

$$\int 6\cos x + 8\sin 2x \;\, dx$$

$y = 6\sin x - 4\cos 2x + c$ and substitute $y = 4$ and $x = \frac{\pi}{6}$ to give

$$4 = 6\sin\frac{\pi}{6} - 4\cos 2(\frac{\pi}{6}) + c$$

$$4 = 6\sin\frac{\pi}{6} - 4\cos\frac{\pi}{3} + c$$

$$4 = 6\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right) + c$$

$c = 3$

So, $y = 6\sin x - 4\cos 2x + 3$

2025 Paper 2 Question 7, (2)

$$\int (3x+2)^7\, dx$$

$$= \frac{(3x+2)^8}{3 \times 8} + c$$

$$= \frac{(3x+2)^8}{24} + c$$

2026 Paper 1 Question 2, (4)

$$\int (15x^{\frac{2}{3}} + \frac{7}{x^2})\ dx$$

$$\int 15x^{\frac{2}{3}} + 7x^{-2}\ dx$$

$$= \frac{15}{\frac{5}{3}}x^{\frac{5}{3}} + \frac{7}{-1}x^{-1} + c$$

$$= 9x^{\frac{5}{3}} - 7x^{-1} + c$$

$$= 9x^{\frac{5}{3}} - \frac{7}{x} + c$$

2026 Paper 1 Question 10, (4)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 6\sin\left(3x - \frac{\pi}{2}\right) dx$$

$$= [-2\cos\left(3x - \frac{\pi}{2}\right)]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= -2\cos\left(3(\frac{\pi}{3}) - \frac{\pi}{2}\right) - [-2\cos\left(3(\frac{\pi}{6}) - \frac{\pi}{2}\right)]$$

$$= -2\cos\left(\pi - \frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right)$$

$$= -2\cos\left(\frac{\pi}{2}\right) + 2\cos(0)$$

$$= 0 + 1$$

$$= 1$$