Higher Mathematics

Graph Transformations - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 4, (2)

The graph of this function has an amplitude of 4, a period of 180°, and is shifted down by 1 unit. So, the solution is A.

2013 Paper 1 Question 11, (2)

$y = -f(x-k)$

Compared to the graph of $y = f(x)$ this graph is inverted and shifted 'k' units to the right. By inspection B is the correct solution.

2014 Paper 1 Question 11, (2)

The co-ordinates given on $f(x)$ are $(2, 3)$ & $(5, 0)$.

Consider transforming these using $y = 2f(x) + 1$.

This transformation tells us to multiply the y co-ordinate by 2 and add 1.

So, $(2, 3)$ becomes $(2, 7)$ and $(5, 0)$ becomes $(5, 1)$.

By inspection graph C has these points present.

2015 Paper 1 Question 4, (3)

The amplitude is 3 so $p = 3$.

The period is 180° so $q = 2$.

The graph has been shifted up by '1' unit after p has been applied, so $r = 1$.

2015 Paper 1 Question 13, (1) (1) (3) (2)

a) $f(x) = 2^x + 3$

Substitute $x = 1$ to give

$f(1) = 2^1 + 3 = 5$

So, $b = 5$.

b) i) The graph would be mirrored in the 45° line $y = x$ with all of the x and y co-ordinates swapped.

ii) The image of $P = (5, 1)$.

The co-ordinate of $Q = (0, 4)$ so the image of $Q = (4, 0)$.

c) $y = 4 - f(x + 1)$

$= -f(x + 1) + 4$

The x co-ordinate of the image of R is 2.

The y co-ordinate of the image of R is $-11 + 4 = -7$.

So, the co-ordinates of the image of R $= (2, -7)$.

2016 Paper 1 Question 15, (3) (1)

a) $f(x) = k(x - a)(x - b)^2$

By inspection of the roots we have

$f(x) = k(x - 4)(x + 5)^2$

Substituting the point (1, 9) gives

$9 = k(1 - 4)(1 + 5)^2$

Rearranging and solving for k gives

$k = -\frac{1}{12}$

b) $g(x) = f(x) - d$

Since d is positive the graph of $g(x)$ is the graph of $f(x)$ moved down by $'d'$ units

By considering the graph of $f(x)$ it must be moved down by at least 9 units to give only one root – in other words to move the point (1, 9) below the x axis. So, $d > 9$

2017 Paper 1 Question 15, (2)

a) The graph of $h(x)$ has been shifted +5 units in the x direction and +3 units in the y direction. So, we have $a = -5, b = 3$.

b) $\int_6^8 h(x)\,dx = \int_1^3 f(x)\,dx + (2.3)$ [2.3 = the area of the rectangle]

$= 4 + 6 = 10$

c) $f'(1) = 6$ tells us that the gradient of the tangent line at $x = 1$ is 6.

By inspection of the symmetry of the graph of $h(x)$ we have $h'(8) = -f'(1) = -6$.

2018 Paper 1 Question 11, (2) (3)

a) $y = 1 - \log_3 x$

$y = -\log_3 x + 1$

The original graph needs to be reflected in the x-axis and moved up 1 unit.

b) $1 - \log_3 x = \log_3 x$

$1 = 2\log_3 x$

$\log_3 x = \frac{1}{2}$

$3^{\frac{1}{2}} = x$

$x = \sqrt{3}$

2019 Paper 1 Question 10, (1) (1)

a) $a = 3$

b) $y = kf(x) + a$

Using the points $(2, -1)$ and $(2, 5)$ for substitution gives

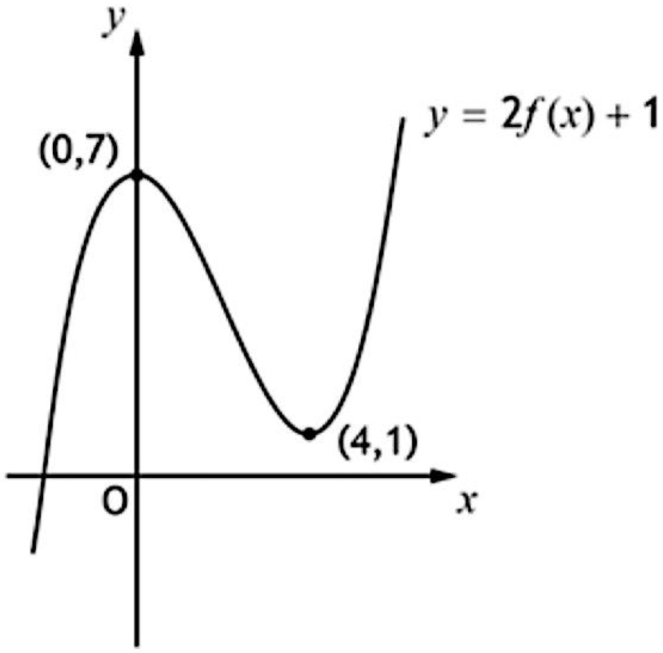
$5 = k(-1) + 3$

$2 = -k$

$k = -2$

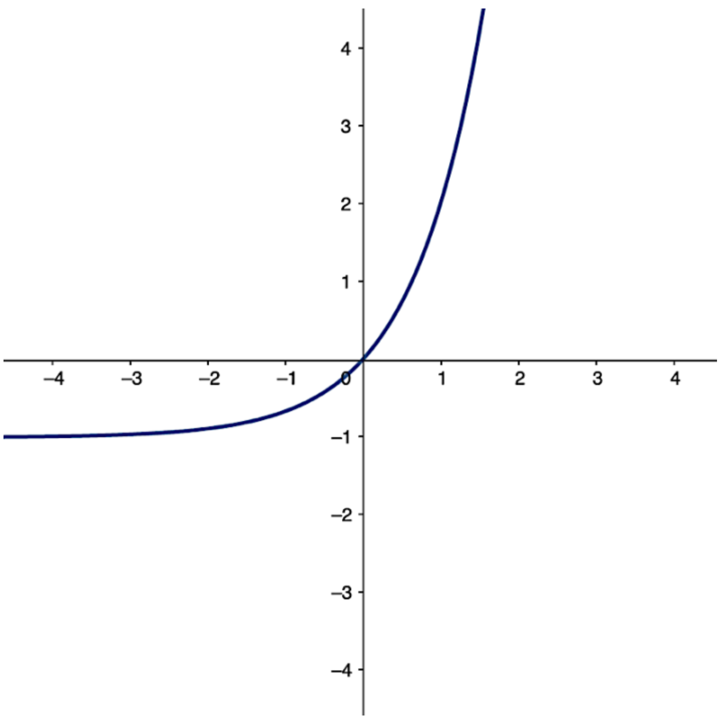
2022 Paper 1 Question 10, (3) (1)

a)



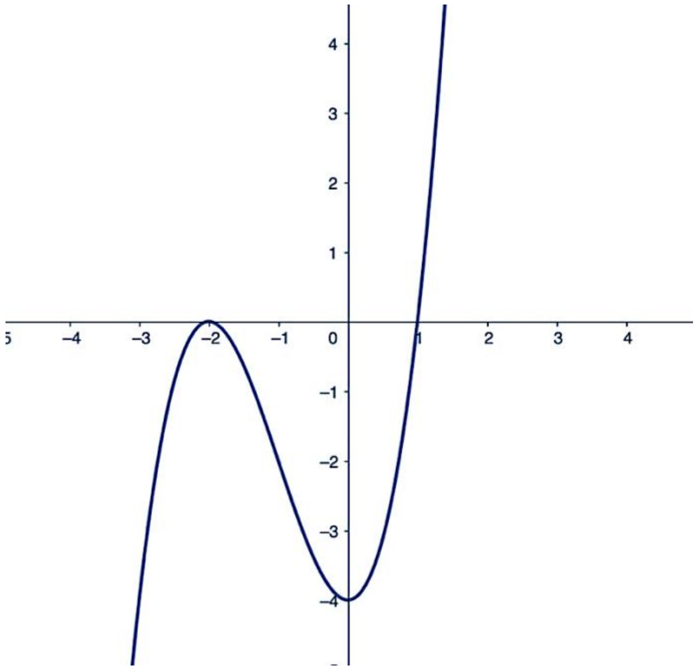
b) $(0, 3)$, $(8, 0)$

2023 Paper 1 Question 9, (3)



2023 Paper 2 Question 4, (2)

$y = 2f(-x)$

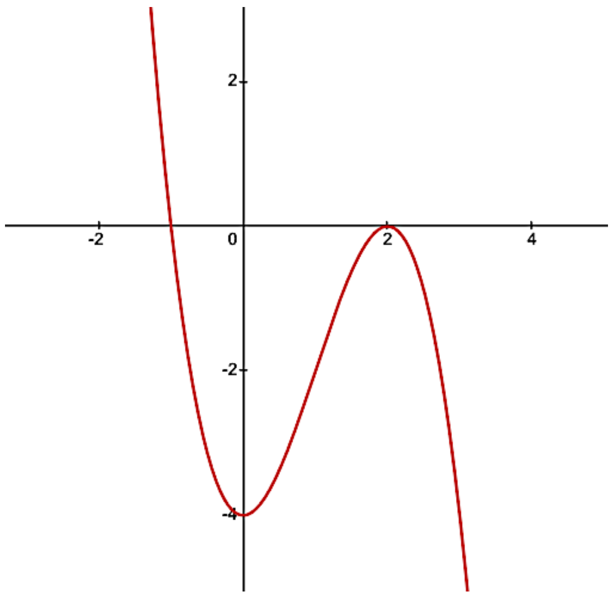


2024 Paper 2 Question 4, (2) (3)

a) The transformation $y = f(x - 4) + 2$ adds 4 to the x co-ordinates and 2 to the y.

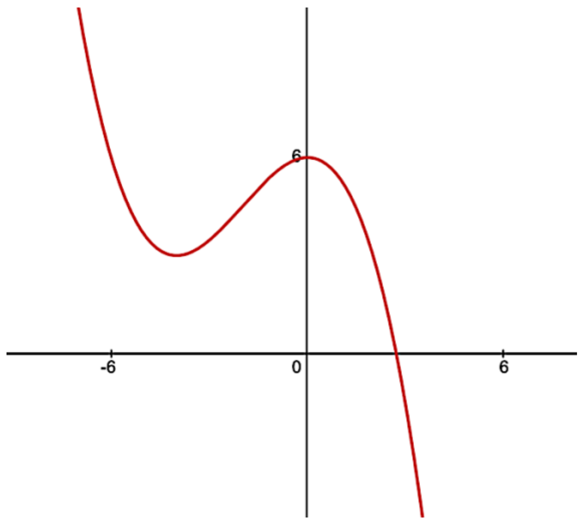
So $(-1, 3)$ becomes $(3, 5)$.

b)



2025 Paper 1 Question 5, (2)

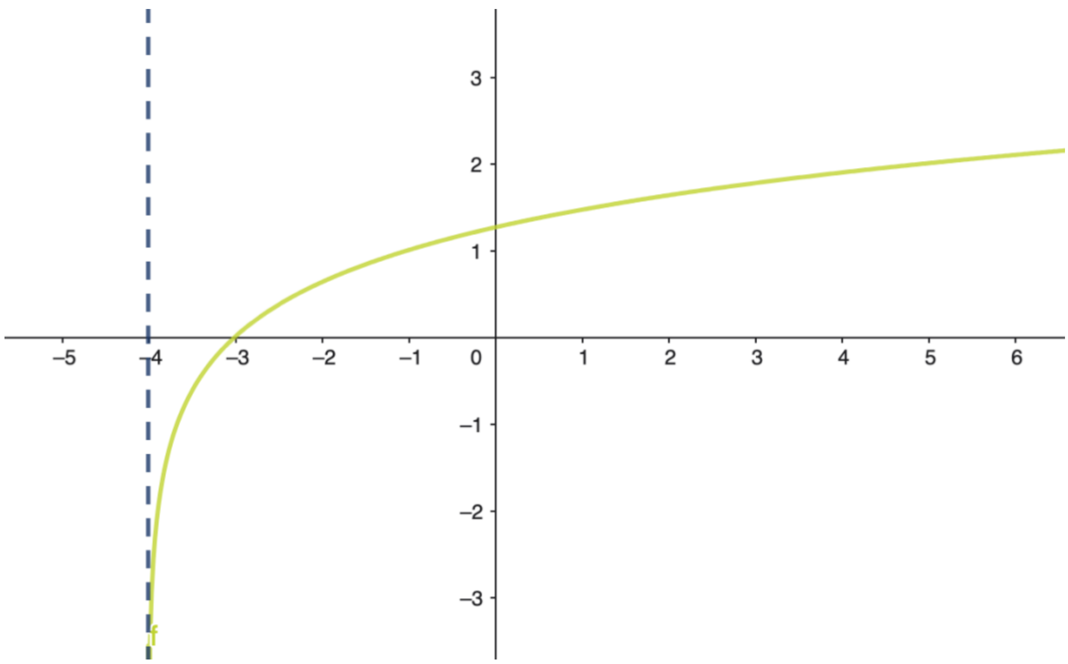
The transformation $y = f(-x) + 3$ divides the x coordinates by -1 and adds 3 to the y coordinates. So, $(0, 3)$ becomes $(0, 6)$, $(4, 0)$ becomes $(-4, 3)$ and $(-2, 0)$ to $(2, 3)$.



2026 Paper 1 Question 12, (2) (2) (1)

a) On the inverse graph the co-ordinates of each point on the graph of f get swapped.

So, the points $(-3, 0)$, $(-1, 1)$ and $(5, 2)$ will be on the graph of $f^{-1}(x)$



b) i) Since the inverse graph passes through $(-3, 0)$ and a basic log graph passes through $(1, 0)$ the graph has been shifted by 4 units to the left. This tells us that $b = 4$. So, $y = \log_a(x + 4)$. Substituting $(5, 2)$ gives:

$2 = \log_a(5 + 4)$

$2 = \log_a 9$

$a^2 = 9$

$a = 3$

b) ii) The domain of the inverse function, $f^{-1}(x)$, is the same as the range of the original function f. The domain of f is the set of y values on the graph of f. By examining the graph of f we see this is $-4 < f(x) < \infty$. So, the domain of $f^{-1}(x)$ is $-4 < f^{-1}(x) < \infty$.

2026 Paper 2 Question 6, (3)

he graph of $y = -f(x) + 1$