Higher Mathematics

Exponential & Logarithm - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 20, (2)

The equation of the line is $\log_3 y = 2x$ since gradient $= 2$ and y-intercept $= 0$

$\log_3 y = 2x$

Rewriting as an exponential gives

$y = 3^{2x}$

2013 Paper 2 Question 5, (4)

$\log_5(3-2x) + \log_5(2+x) = 1$

$\log_5[(3-2x)(2+x)] = 1$

$5^1 = (3-2x)(2+x)$

$5 = 6 - x - 2x^2$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = -1, x = \dfrac{1}{2}$

2013 Paper 2 Question 9, (4) (3)

a) $p_t = p_0 e^{-kt}$

Since the concentration has halved $p_t = \frac{p_0}{2}$

$\frac{p_0}{2} = p_0 e^{-25k}$

$0.5 = e^{-25k}$

$\log_e 0.5 = \log_e e^{-25k}$

$\log_e 0.5 = -25k$

$k = \frac{\log_e 0.5}{-25}$

$k = 0.028$ to 2 significant figures

b) $p_t = p_0 e^{-kt}$

Let $t = 80$ and $k = 0.028$. Substitute to give

$p_t = p_0 e^{-0.028 \, x \, 80}$

$p_t = p_0 e^{-2.24}$

$p_t = 0.1065 \, p_0$

Rounding gives

$p_t = 0.11 \, p_0$

So, p_t is 11% of p_0

Therefore, the concentration has decreased by 89%.

2014 Paper 1 Question 3, (2)

$\log_4 12 - \log_4 x = \log_4 6$

$\log_4 12 - \log_4 6 = \log_4 x$

$\log_4 \frac{12}{6} = \log_4 x$

$\log_4 2 = \log_4 x$

$x = 2$

2014 Paper 1 Question 20, (2)

$2 - \log_5 \frac{1}{25} = 2 - (\log_5 1 - \log_5 25) = 2 - 0 + 2 = 4$

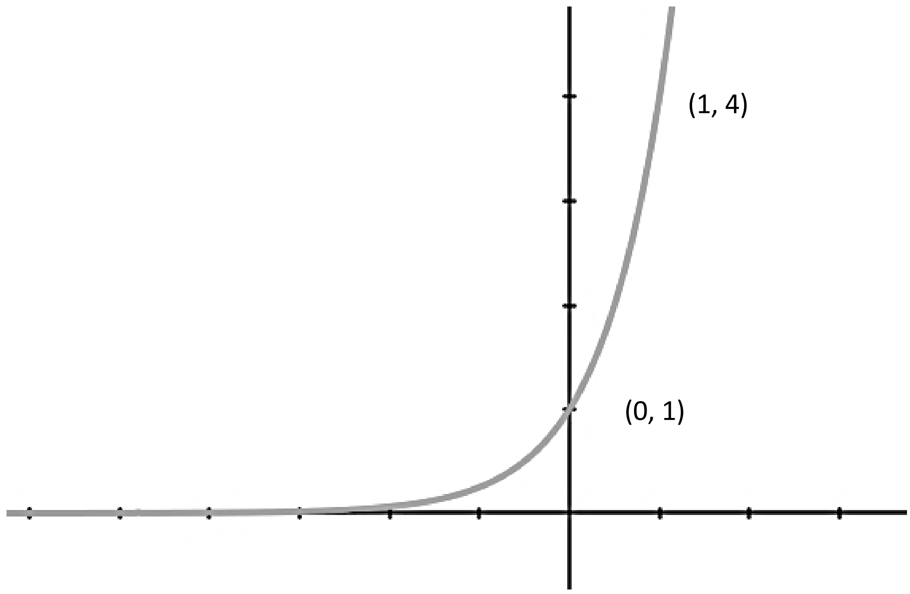
2015 Paper 1 Question 6, (3)

$$\begin{aligned}\log_6 12 + \frac{1}{3}\log_6 27 &= \log_6 12 + \log_6 27^{\frac{1}{3}} \\ &= \log_6 12 + \log_6 3 \\ &= \log_6 36 \\ &= 2\end{aligned}$$

2016 Paper 1 Question 10, (2)

The point $(1, 0)$ on the graph of $f(x)$ is transformed to $(0, 1)$ on the graph of $f^{-1}(x)$.

The point $(4, 1)$ on the graph of $f(x)$ is transformed to $(1, 4)$ on the graph of $f^{-1}(x)$.



2016 Paper 1 Question 14, (1) (5)

a) $\log_5 25 = 2$ since $5^2 = 25$

b) $\log_4 x + \log_4(x-6) = \log_5 25$

$\log_4 x + \log_4(x-6) = 2$

$\log_4 x(x-6) = 2$

Rewriting as an exponential gives

$x(x-6) = 4^2$

$x^2 - 6x = 16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = -2, x = 8$

2016 Paper 2 Question 6, (1) (4)

a) $B(t) = 200e^{0.107t}$

At the start $t = 0$ giving $B(0) = 200 \cdot e^0 = 200$.

b) When the number doubles there are 400 bacteria.

Let $B(t) = 400$

$400 = 200e^{0.107t}$

$2 = e^{0.107t}$

$\log_e 2 = \log_e e^{0.107t}$

$\log_e 2 = 0.107t$

$t = \frac{\log_e 2}{0.107} = 6.48$ hours.

2017 Paper 1 Question 12, (3)

$\log_a 36 - \log_a 4 = \frac{1}{2}$

$\log_a \left(\frac{36}{4}\right) = \frac{1}{2}$

$\log_a 9 = \frac{1}{2}$

Rewriting as an exponential to give

$a^{\frac{1}{2}} = 9$

$a = 9^2 = 81$

2018 Paper 1 Question 6, (3)

$\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{\frac{1}{3}}$

$= \log_5 250 - \log_5 2$

$= \log_5 \frac{250}{2}$

$= \log_5 125$

$= 3$

2018 Paper 1 Question 11, (2) (3)

a) $y = 1 - \log_3 x$

$y = -\log_3 x + 1$

The original graph needs to be reflected in the x-axis and moved up 1 unit.

b) $1 - \log_3 x = \log_3 x$

$1 = 2\log_3 x$

$\log_3 x = \frac{1}{2}$

$3^{\frac{1}{2}} = x$

$x = \sqrt{3}$

2018 Paper 2 Question 11, (4) (2)

a) $P = 100(1 - e^{-kt})$

$50 = 100(1 - e^{-kt})$

$0.5 = 1 - e^{-kt}$

$e^{-kt} = 0.5$

$log_e e^{3k} = log_e 0.5$

$3k = ln\, 0.5$

$k = \frac{ln\, 0.5}{3}$

$k = -0.231$

b) $P = 100(1 - e^{-0.231t})$

$= 100(1 - e^{-0.231\, x\, 5})$

$= 100 - 100e^{-1.155}$

$= 100 - 31.51$

$= 68.49$

$= 68.5$

2019 Paper 1 Question 14, (3) (3)

a) $\log_{10} 4 + 2\log_{10} 5$

$= \log_{10} 4 + \log_{10} 5^2$

$= \log_{10} 4 + \log_{10} 25$

$= \log_{10} 100$

$= 2$

b) $\log_2(7x-2) - \log_2 3 = 5$

$$\log_2\left(\frac{7x-2}{3}\right) = 5$$

$$2^5 = \frac{7x-2}{3}$$

$$32 = \frac{7x-2}{3}$$

$$96 = 7x - 2$$

$$7x = 98$$

$$x = 14$$

2019 Paper 2 Question 9, (1) (4)

a) Initially $t = 0$ to give

$P_0 = 120e^{-0.0079\,x\,0} = 120e^0 = 120$

Initial electrical power is 120 Watts.

b) 15% reduction $= 120\ x\ 0.85 = 102$

Substitute 102 to give

$102 = 120e^{-0.0079t}$

$\frac{102}{120} = e^{-0.0079t}$

$0.85 = e^{-0.0079t}$

$\log_e 0.85 = \log_e e^{-0.0079t}$

$\ln 0.85 = -0.0079t$

$t = \frac{\ln 0.85}{-0.0079}$

$= 20.57$

So, it would take 20.57 years.

2022 Paper 1 Question 2, (3)

$2\log_3 6 - \log_3 4$

$= \log_3 6^2 - \log_3 4$

$= \log_3 36 - \log_3 4$

$= \log_3 \frac{36}{4}$

$= \log_3 9$

$= 2$

2022 Paper 1 Question 8, (4)

$\log_6 x + \log_6(x+5) = 2$

$= \log_6 x\,(x+5) = 2$

$= \log_6(x^2 + 5x) = 2$

Rearrange as an exponential to give

$x^2 + 5x = 6^2$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x + 9 = 0$ and $x - 4 = 0$

$x = -9$ and $x = 4$

Since $x > 0$ the solution is $x = 4$

2022 Paper 2 Question 10, (1) (4)

a) $P = 4.99087\,(42.5 - 24.55)^{1.81}$

$= 929$

b) $850 = 0.188807\,(600 - 210)^k$

$(600-210)^k = \frac{850}{0.188807}$

$(390)^k = \frac{850}{0.188807}$

$\ln(390)^k = \ln\frac{850}{0.188807}$

$k\ln 390 = \ln\frac{850}{0.188807}$

$k = \frac{\ln\left(\frac{850}{0.188807}\right)}{\ln 390} = 1.41$

2023 Paper 1 Question 3, (3)

$\log_5 x - \log_5 3 = 2$

$\log_5 \frac{x}{3} = 2$

$\frac{x}{3} = 5^2 = 25$

$x = 75$

2023 Paper 1 Question 7, (2) (1)

a) $\log_2 5 + \log_2 \frac{1}{40}$

$= \log_2 \frac{5}{40}$

$= \log_2 \frac{5}{40}$

$= \log_2 \frac{1}{8}$

$= \log_2 1 - \log_2 8$

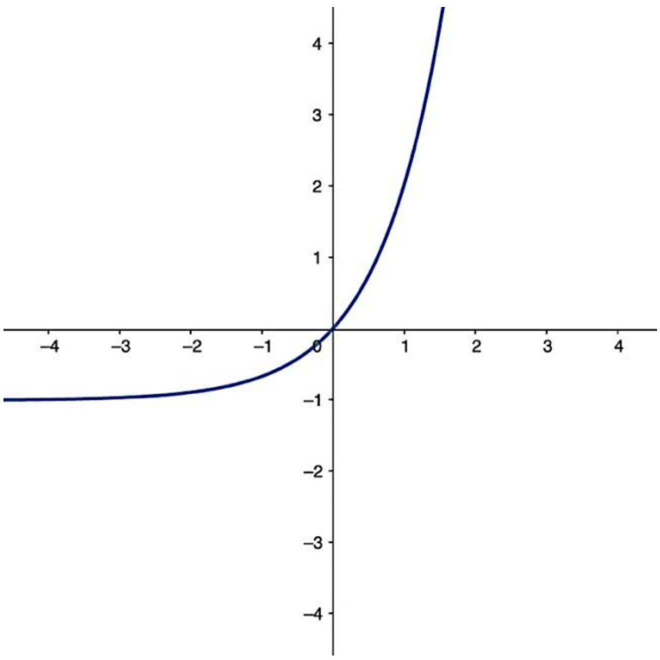
$= 0 - 3$

$= -3$

b) The logarithm function is negative for values between 0 and 1 (consider the graph).

So, $0 < a < 1$

2023 Paper 1 Question 9, (3)



2023 Paper 2 Question 13, (1) (3)

a) $c_t = 11e^{-0.0053t}$

Substitute $t = 30$ to give

$c_{30} = 11e^{-0.0053\, x\, 30}$

$= 9.38$

Concentration is 9.38 mg/l.

b) $c_t = 11e^{-0.0053t}$

Substitute $c_t = 0.66$ to give

$0.66 = 11e^{-0.0053t}$

$\frac{0.66}{11} = e^{-0.0053t}$

$\ln \frac{0.66}{11} = -0.0053t$

$t = (\ln \frac{0.66}{11})/-0.0053$

$t = 530.8$

So, the time is 538.8 minutes.

2024 Paper 1 Question 9, (3)

$\log_a 5 + \log_a 80 - 2\log_a 10$

$= \log_a(5 \times 80) - \log_a 10^2$

$= \log_a 400 - \log_a 100$

$= \log_a \frac{400}{100}$

$= \log_a 4$

2024 Paper 2 Question 11, (1) (4)

a) $N = 6.8\, e^{kt}$

Substitute $t = 0$ to give

$N = 6.8\, e^{k(0)} = 6.8$

So, there are 6.8 million cars.

b) For 2030 the t value is 10. Substitute this and 125 into the equation to give

$125 = 6.8\,e^{10k}$

$e^{10k} = \frac{125}{68}$

$\ln e^{10k} = \ln\frac{125}{68}$

$10k = \ln\frac{125}{68}$

$k = \ln\frac{125}{68} \div 10$

$= 0.061$

2025 Paper 1 Question 4, (3)

$3\log_3 2 + \log_3 \dfrac{1}{24}$

$= \log_3 2^3 + \log_3 \dfrac{1}{24}$

$= \log_3 8 + \log_3 \dfrac{1}{24}$

$= \log_3 \dfrac{8}{24}$

$= \log_3 \frac{1}{3} = -1$ since $3^{-1} = \frac{1}{3}$

2025 Paper 1 Question 8, (3)

$\log_a 75 = 2 + \log_a 3$

$\log_a 75 - \log_a 3 = 2$

$\log_a \dfrac{75}{3} = 2$

$\log_a 25 = 2$

$a = 5$

2025 Paper 2 Question 13, (1) (4)

a) Substitute $t = 0$ for initial mass to give

$M = 150\, e^{-0.0054 \times 0}$

$= 150\, e^{0}$

$= 150 \times 1$

$= 150$

So, the initial mass is 150 micrograms.

b) $M = 150\, e^{-0.0054t}$

Substitute 120 for M to give

$120 = 150\, e^{-0.0054t}$

$e^{-0.0054t} = \frac{120}{150}$

$\ln(e^{-0.0054t}) = \ln\frac{120}{150}$

$-0.0054t = \ln\frac{120}{150}$

$t = \ln\frac{120}{150} \div -0.0054$

$t = 41.32$

So, the time is 41.32 years

2026 Paper 1 Question 8, (4)

$\log_2 x + \log_2(x + 2) = 3$

$\log_2(x^2 + 2x) = 3$

$x^2 + 2x = 2^3$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$ and $x - 2 = 0$

$x = -4$ and $x = 2$

Since $x > 0$ we can discard $x = -4$ so $x = 2$

2026 Paper 2 Question 14, (1) (4)

a) $T = 800\,e^{-0.124t} + 25$

Substitute $t = 0$ to give

$T = 800\,e^{-0.124 \times 0} + 25$

$= 800 + 25$

$= 825$ Celsius

b) Substitute $T = 150$ to give

$150 = 800\,e^{-0.124t} + 25$

$800\,e^{-0.124t} = 125$

$e^{-0.124t} = \frac{125}{800}$

$\ln(e^{-0.124t}) = \ln\frac{125}{800}$

$-0.124t = \ln\frac{125}{800}$

$t = \ln\frac{125}{800} \div -0.124$

$= 14.97$

The time taken is 14.97 seconds.