Higher Mathematics

Exponential & Logarithm - Questions

Marks are indicated in brackets after each question number

2013 Paper 2 Question 9, (4) (3)

The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?

2013 Paper 2 Question 5, (4)

Solve the equation

$$\log_5(3-2x) + \log_5(2+x) = 1, \text{ where } x \text{ is a real number.}$$

2014 Paper 1 Question 3, (2)

If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?

2014 Paper 1 Question 20, (2)

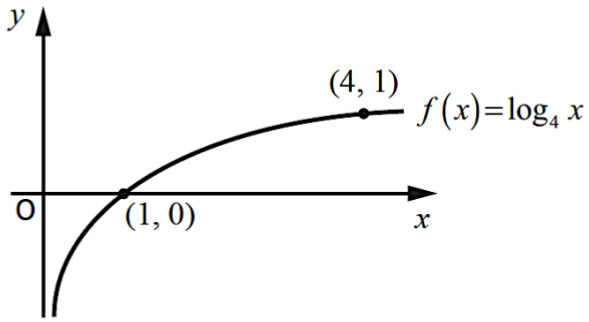
Evaluate $2 - \log_5 \frac{1}{25}$.

2015 Paper 1 Question 6, (3)

Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.

2016 Paper 1 Question 10, (2)

The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function.

2016 Paper 1 Question 14, (1) (5)

(a) Evaluate $\log_5 25$.

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$.

2016 Paper 2 Question 6, (1) (4)

Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200\, e^{0{\cdot}107t},$$

where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study.

(b) Calculate the time taken for the number of bacteria to double.

2017 Paper 1 Question 12, (3)

Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a.

2018 Paper 1 Question 6, (3)

Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$.

2018 Paper 2 Question 11, (4) (2)

A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, $P\%$, of customers who wait for less than t minutes, can be modelled by

$$P = 100\left(1 - e^{kt}\right), \text{ where } k \text{ is a constant.}$$

(a) If 50% of customers wait for less than 3 minutes, determine the value of k.

(b) Calculate the percentage of customers who wait for 5 minutes or longer.

s2019 Paper 1 Question 14, (3) (3)

(a) Evaluate $\log_{10} 4 + 2\log_{10} 5$.

(b) Solve $\log_2(7x - 2) - \log_2 3 = 5, \quad x \geq 1$.

2019 Paper 2 Question 9, (1) (4)

Electricity on a spacecraft can be produced by a type of nuclear generator.

The electrical power produced by this generator can be modelled by

$$P_t = 120e^{-0.0079t}$$

where P_t is the electrical power produced, in watts, after t years.

(a) Determine the electrical power initially produced by the generator.

(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%.

2022 Paper 1 Question 2, (3)

Evaluate $2\log_3 6 - \log_3 4$.

2022 Paper 1 Question 8, (4)

Solve $\log_6 x + \log_6 (x+5) = 2$, where $x > 0$.

2022 Paper 2 Question 10, (1) (4)

The heptathlon is an athletics contest made up of seven events.

Athletes score points for each event.

In the 200 metres event, the points are calculated using the formula

$$P = 4.99087\left(42.5 - T\right)^{1.81}$$

where P is the number of points awarded, and T is the athlete's time, in seconds.

(a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event.

In the long jump event, the points are calculated using the formula

$$P = 0.188807\left(D - 210\right)^{k}$$

where P is the number of points awarded, D is the distance jumped, in centimetres, and k is a constant.

(b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k.

2023 Paper 1 Question 3, (3)

Solve $\log_5 x - \log_5 3 = 2$.

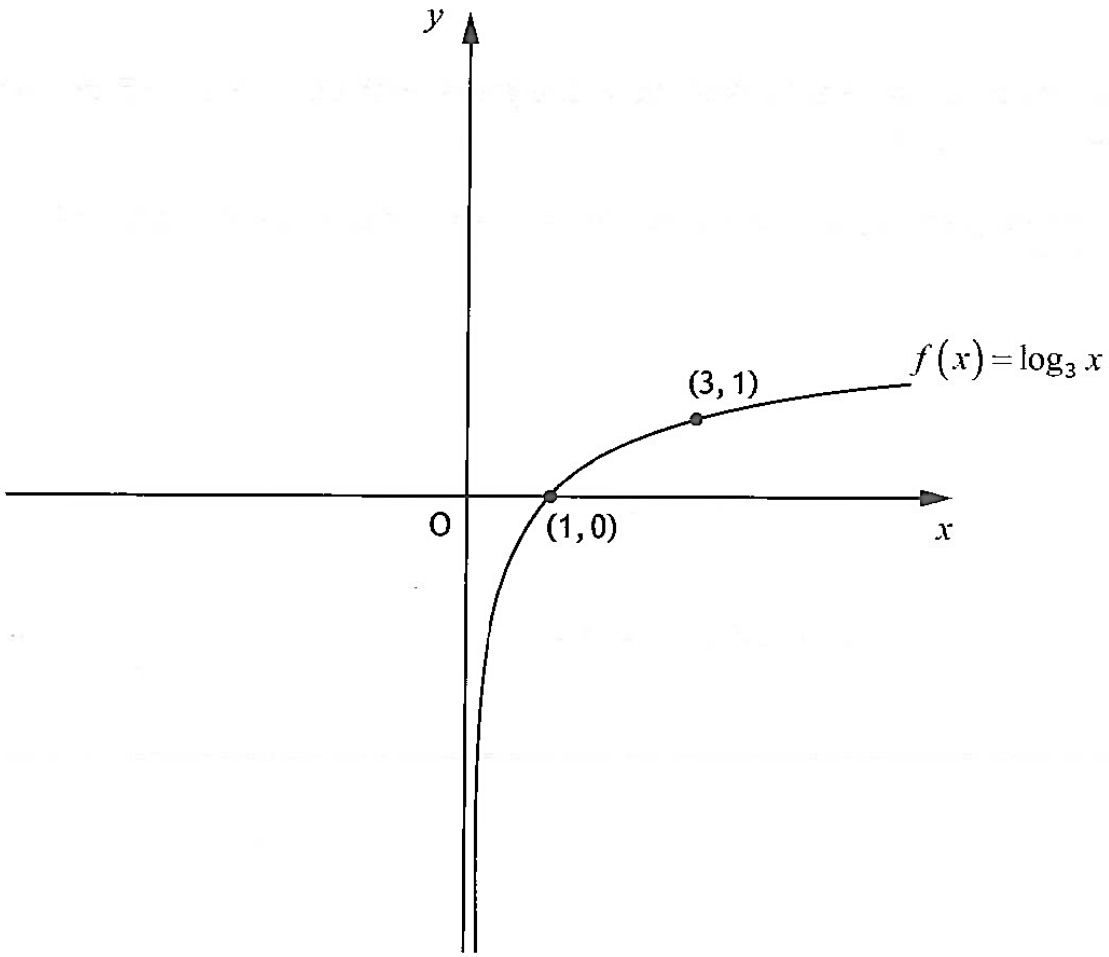
2023 Paper 1 Question 7, (2) (1)

(a) Evaluate $\log_2 5 + \log_2 \frac{1}{40}$.

(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a.

2023 Paper 1 Question 9, (3)

The diagram shows the graph of the function $f(x) = \log_3 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of $y = f^{-1}(x) - 1$.

2023 Paper 2 Question 13, (1) (3)

A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by

$$C_t = 11e^{-0.0053t}$$

where:

- t is the time, in minutes, since the dose of medicine was given
- C_t is the concentration of the medicine, in mg/l, at time t.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given.

The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

(b) Calculate the time taken for this dose of the medicine to become ineffective.

2024 Paper 1 Question 9, (3)

Express $\log_a 5 + \log_a 80 - 2\log_a 10$ in the form $\log_a k$ where k is a positive integer.

2024 Paper 2 Question 11, (1) (4)

The number of electric vehicles worldwide can be modelled by

$$N = 6.8e^{kt}$$

where:

- N is the estimated number of vehicles in millions
- t is the number of years since the end of 2020
- k is a constant.

(a) Use the model to estimate the number of electric vehicles worldwide at the end of 2020.

At the end of 2030, it is estimated there will be 125 million electric vehicles worldwide.

(b) Determine the value of k.

2025 Paper 1 Question 4, (3)

Evaluate $3\log_3 2 + \log_3 \frac{1}{24}$.

2025 Paper 1 Question 8, (3)

Given that $\log_a 75 = 2 + \log_a 3$, $a > 0$, find the value of a.

2025 Paper 2 Question 13, (1) (4)

A radioactive substance, which has been collected, decays over time.

The mass of the radioactive substance remaining is modelled by

$$M = 150e^{-0.0054t}$$

where M is the mass, in micrograms, t years after the radioactive substance was collected.

(a) Determine the initial mass of the radioactive substance.

(b) Calculate the time taken for the mass of the radioactive substance to decay to 120 micrograms.

2026 Paper 1 Question 8, (4)

Solve $\log_2 x + \log_2 (x+2) = 3$, where $x > 0$.

2026 Paper 2 Question 14, (1) (4)

During a metal-working process a heated steel rod is cooled by dropping it into a container of oil.

The temperature of the rod is given by

$$T = 800e^{-0.124t} + 25,$$

where T is the temperature of the rod, in degrees Celsius, t seconds after it is dropped into the oil.

(a) Determine the temperature of the rod as it is dropped into the oil.

(b) Calculate the time taken for the temperature of the rod to cool to 150 °C.