



Higher Mathematics

Domain - Solutions

Marks are indicated in brackets after each question number

2013 Paper 1 Question 13, (2)

$$\text{Let } x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x = 3, x = 4$$

So, $x = 3, x = 4$ cannot be in the domain of $f(x)$.

2014 Paper 1 Question 12, (2)

$$f(x) = \frac{6x}{x^2 + 6x - 16}$$

Restrictions on the domain of $f(x)$ occur where $x^2 + 6x - 16 = 0$ since we cannot divide by 0.

$$\text{Let } x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

$$x = -8, x = 2$$

So, the restrictions on the domain of f are $x = -8, x = 2$.

2015 Paper 2 Question 2, (2) (3) (2)

$$\begin{aligned} \text{a) } f(g(x)) &= f((1+x)(3-x) + 2) \\ &= 10 + (1+x)(3-x) + 2 \end{aligned}$$

Simplifying gives

$$f(g(x)) = 15 + 2x - x^2$$

$$\begin{aligned} \text{b) } f(g(x)) &= -x^2 + 2x + 15 \\ &= -(x^2 - 2x) + 15 \end{aligned}$$

Completing the square of the inside of the bracket

$$= -[(x - 1)^2 - 1] + 15 = -(x - 1)^2 + 16$$



$$\text{c) } h(x) = \frac{1}{f(g(x))} = \frac{1}{-(x-1)^2+16}$$

Restrictions on the domain of $h(x)$ occur where $-(x-1)^2 + 16 = 0$.

$$\text{Let } -(x-1)^2 + 16 = 0$$

$$(x-1)^2 = 16$$

$$x-1 = 4 \quad x-1 = -4$$

$$x = 5 \quad x = -3$$

So, $x = -3, x = 5$ cannot be in the domain of $h(x)$.

2019 Paper 1 Question 12, (2) (1)

$$\text{a) } f(g(x)) = f(5-x) = \frac{1}{\sqrt{5-x}}$$

b) $f(g(x))$ is undefined where $5-x \leq 0$ since you cannot square root a negative and you cannot divide by zero.

$$\text{So, } 5-x \leq 0$$

$$5 \leq x$$

$$x \geq 5$$

2019 Paper 2 Question 8, (3) (1)

$$\text{a) } f(x) = \sqrt[3]{x} + 8$$

$$y = \sqrt[3]{x} + 8$$

$$y-8 = \sqrt[3]{x}$$

$$x = (y-8)^3$$

$$\text{So, } f^{-1}(x) = (x-8)^3$$

b) The domain of $f^{-1}(x)$ is the range of $f(x)$

$$f(1) = 9$$

$$f(1000) = 18$$

So, the range of $f(x)$ is $9 \leq x \leq 18$.

Hence, the domain of $f^{-1}(x)$ is $9 \leq x \leq 18, x \in \mathbb{R}$.



2024 Paper 2 Question 8, (2) (2)

$$\begin{aligned}\text{a) } f(g(x)) &= f(x + 1) \\ &= 2(x + 1)^2 - 18 \\ &= 2(x^2 + 2x + 1) - 18 \\ &= 2x^2 + 4x + 2 - 18 \\ &= 2x^2 + 4x - 16\end{aligned}$$

$$\text{b) } \frac{1}{f(g(x))} = \frac{1}{2x^2 + 4x - 16}$$

The function is undefined where the denominator, $2x^2 + 4x - 16$, equals zero.

$$2x^2 + 4x - 16 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$x = -4, x = 2$$