# Higher Mathematics

## Differentiation - Solutions

Marks are indicated in brackets after each question number

### 2013 Paper 1 Question 2, (2)

$y = x^2 - 4x + 7$

$\frac{dy}{dx} = 2x - 4$

Gradient of tangent at $(5, 12) = 2 \cdot 5 - 4 = 6$

Using $y - b = m(x - a)$ gives

$y - 12 = 6(x - 5)$

$y - 12 = 6x - 30$

$y = 6x - 28$

### 2013 Paper 1 Question 18, (2)

$y = \sin(x^2 - 3)$

$\frac{dy}{dx} = \cos(x^2 - 3) \cdot 2x$

$\quad = 2x\cos(x^2 - 3)$

### 2014 Paper 1 Question 8, (2)

Let $f(x) = (4 - 9x^4)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}} \cdot (-36x^3)$

$\quad = -18x^3(4 - 9x^4)^{\frac{1}{2}}$

### 2014 Paper 1 Question 9, (2)

This approach uses the derivative to find max values but there are other methods.

Let $f(x) = 5\sin 2x + 5\sqrt{3}\cos 2x$

$f'(x) = 10\cos 2x - 10\sin 2x$

For maximum values $f'(x) = 0$

$10\cos 2x - 10\sin 2x = 0$

$\cos 2x - \sin 2x = 0$

$\cos 2x = \sin 2x$

$1 = \dfrac{\sin 2x}{\cos 2x} = \tan 2x$

From exact values $2x = \frac{\pi}{6}$

So, the maximum value occurs where $2x = \frac{\pi}{6}$

Substituting $2x = \frac{\pi}{6}$ gives

$f(x) = 5\sin\frac{\pi}{6} + 5\sqrt{3}\cos\frac{\pi}{6} = \frac{5}{2} + \frac{15}{2} = 10$

**2014 Paper 2 Question 2, (4)**

$y = x^4 - 2x^3 + 5$

$\dfrac{dy}{dx} = 4x^3 - 6x^2$

When $x = 2, \frac{dy}{dx} = 32 - 24 = 8$

So, the gradient of the tangent at $x = 2$ is 8.

When $x = 2, y = 2^4 - 2 \cdot 2^3 + 5 = 5$

Using $y - b = m(x - a)$ with $(2, 5)$ gives

$y - 5 = 8(x - 2)$

Rearranging gives

$y = 8x - 11$

**2015 Paper 1 Question 2, (4)**

$y = 2x^3 + 3$

$\dfrac{dy}{dx} = 6x^2$

When $x = -2, \frac{dy}{dx} = 6 \cdot (-2)^2 = 24$

So, gradient of tangent at $x = -2$ is 24.

When $x = -2, y = 2 \cdot (-2)^3 + 3 = -13$

So, $(-2, -13)$ lies on the curve.

Using $y - b = m(x - a)$ we have

$y - (-13) = -2(x - (-13))$

$y + 13 = -2x - 26$

$y = -2x - 39$

**2015 Paper 1 Question 7, (4)**

$f(x) = \sqrt{x}(3x - \frac{2}{x\sqrt{x}})$

$= x^{\frac{1}{2}}(3x - \frac{2}{x \cdot x^{\frac{1}{2}}})$

$= x^{\frac{1}{2}}(3x - \frac{2}{x^{\frac{3}{2}}})$

$= x^{\frac{1}{2}}(3x - 2x^{-\frac{3}{2}})$

$= 3x^{\frac{3}{2}} - 2x^{-1}$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 2x^{-2} = \frac{9}{2}x^{\frac{1}{2}} + \frac{2}{x^2}$

$f'(4) = \left(\frac{9}{2} \cdot 2\right) + \frac{2}{16} = 9 + \frac{1}{8} = \frac{73}{8}$

**2016 Paper 1 Question 2, (3)**

$y = 12x^3 + 8\sqrt{x}$

$y = 12x^3 + 8x^{\frac{1}{2}}$

$\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$

**2016 Paper 2 Question 10, (2) (1)**

**a)** $y = (x^2 + 7)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^2 + 7)^{-\frac{1}{2}} \cdot 2x$

$= x(x^2 + 7)^{-\frac{1}{2}}$

$= \frac{x}{(x^2 + 7)^{\frac{1}{2}}}$

$= \frac{x}{\sqrt{x^2 + 7}}$

**b)** $\int \frac{4x}{\sqrt{x^2+7}}\,dx$

$= 4\int \frac{x}{\sqrt{x^2+7}}\,dx$

$= 4(x^2+7)^{\frac{1}{2}}$ using the result from **a)**

## 2017 Paper 1 Question 3, (2)

$y = (4x-1)^{12}$

$\frac{dy}{dx} = 12(4x-1)^{11} \cdot 4$

$= 48(4x-1)^{11}$

## 2017 Paper 1 Question 8, (3)

$$d(t) = \frac{1}{2t} = \frac{t^{-1}}{2} = \frac{1}{2}t^{-1}$$

$$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$$

$$d'(5) = -\frac{1}{2 \cdot 5^2} = -\frac{1}{50}$$

So, the rate of change of $d(t)$ when $t = 5$ is $-\frac{1}{50}$.

## 2018 Paper 1 Question 3, (3)

$h(x) = 3\cos(2x)$

$h'(x) = -6\sin(2x)$

$h'\left(\frac{\pi}{6}\right) = -6\sin\left(2\,x\,\frac{\pi}{6}\right)$

$= -6\sin\left(\frac{\pi}{3}\right)$

$= -6\,x\,\frac{\sqrt{3}}{2}$

$= -3\sqrt{3}$

**2018 Paper 1 Question 7, (1) (3) (4)**

**a)** $(0, 5)$

**b)** $y = x^3 - 3x^2 + 2x + 5$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$m_{tan} = 3(0)^2 - 6(0) + 2$

$= 2$

$y = 2x + 5$

**c)** $2x + 5 = x^3 - 3x^2 + 2x + 5$

$x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

$x = 0, x = 3$

When $x = 3, y = (2\ x\ 3) + 5 = 11$. Giving $Q = (3, 11)$.

**2018 Paper 1 Question 15, (5)**

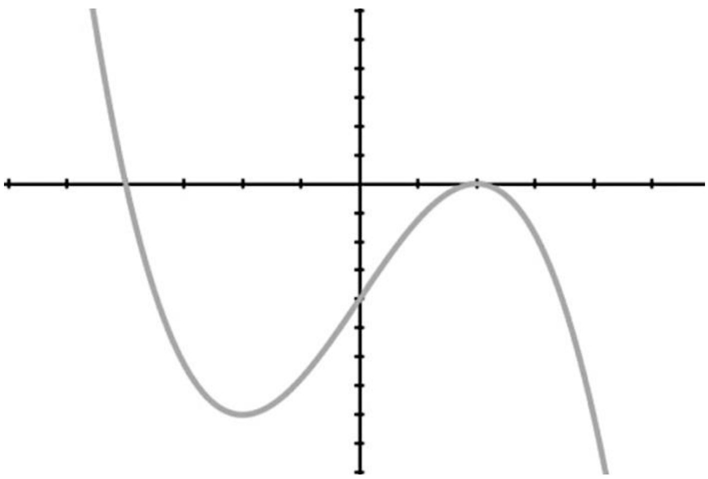
Since $(x + 4)$ is a factor, $x = -4$ is a root.

Since $x = 2$ is a repeated root, $(x - 2)$ is a repeated factor.

So, the equation of the function can be written $f(x) = k(x + 4)(x - 2)(x - 2)$.

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$.

Considering all of the above information and assuming a $k$ value (which cannot be determined from the information) a possible graph is:



**2018 Paper 2 Question 3, (3)**

$f(x) = x^3 - 7x - 6$

$f'(x) = 3x^2 - 7$

$f'(2) = 5$

Since $f'(2) > 0$, the function is increasing.

**2018 Paper 2 Question 9, (6)**

$$P(x) = 2x + \frac{128}{x}$$

$$= 2x + 128x^{-1}$$

$$P'(x) = 2 - 128x^{-2} = 2 - \frac{128}{x^2}$$

For minimum values, $P'(x) = 0$

$$2 - \frac{128}{x^2} = 0$$

$2x^2 - 128 = 0$

$x^2 = 64$

$x = -8, x = 8$

Discard $x = -8$ since $x$ cannot be negative

| $x$ | 1 | 8 | 10 |
|---|---|---|---|
| $P'(x)$ | $-$ | 0 | $+$ |
| *Shape* | | | |

Substitute $x = 8$ to give

$$P(x) = (2\ x\ 8) + \frac{128}{8}$$

$$= 16 + 16 = 32$$

**2019 Paper 1 Question 1, (4)**

$$y = \frac{1}{2}x^4 - 2x^3 + 6$$

$$\frac{dy}{dx} = 2x^3 - 6x^2$$

**2019 Paper 1 Question 6, (3)**

$$y = \frac{1}{(1-3x)^5}$$

$$= (1-3x)^{-5}$$

$$\frac{dy}{dx} = -5(1-3x)^{-6} \cdot (-3)$$

$$= 15(1-3x)^{-6}$$

**2022 Paper 1 Question 4, (3)**

$$y = \sqrt{x^3} - 2x^{-1}$$

$$y = x^{\frac{3}{2}} - 2x^{-1}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-2}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + \frac{2}{x^2}$$

**2022 Paper 1 Question 12, (3)**

$$f(x) = 4\sin(3x - \frac{\pi}{3})$$

$$f'(x) = 12\cos(3x - \frac{\pi}{3})$$

$$f'\left(\frac{\pi}{6}\right) = 12\cos(3(\frac{\pi}{6}) - \frac{\pi}{3})$$

$$= 12\cos(\frac{\pi}{2} - \frac{\pi}{3})$$

$$= 12\cos(\frac{\pi}{6})$$

$$= 12 \cdot \frac{\sqrt{3}}{2}$$

$$= 6\sqrt{3}$$

**2023 Paper 1 Question 1, (3)**

$y = x^{\frac{5}{3}} - \dfrac{10}{x^4}$

$y = x^{\frac{5}{3}} - 10x^{-4}$

$\dfrac{dy}{dx} = \dfrac{5}{3}x^{\frac{2}{3}} + 40x^{-5}$

$\dfrac{dy}{dx} = \dfrac{5}{3}x^{\frac{2}{3}} + \dfrac{40}{x^5}$

**2023 Paper 2 Question 2, (4)**

$y = 2x^5 - 3x$

$\dfrac{dy}{dx} = 10x - 3$

Substitute $x = 1$ for gradient to give $m = 10(1) - 3 = 7$.

When $x = 1, y = 2(1)^5 - 3(1) = -1$

So, the point of tangency is $(1, -1)$.

Using $y - b = m(x - a)$ with $(1, -1)$ gives

$y - (-1) = 7(x - 1)$

$y = 7x - 8$

**2023 Paper 2 Question 5, (3)**

$f(x) = (3 - 2x)^4$

$f'(x) = 4(3 - 2x)^3 \cdot -2$

$= -8(3 - 2x)^3$

$f'(4) = -8(3 - 2(4))^3$

$= -8(-5)^3$

$= 1000$

So, the rate of change is 1000.

**2023 Paper 2 Question 10, (4)**

$f(x) = 2x^3 + 9x^2 - 24x + 6$

$f'(x) = 6x^2 + 18x - 24$

Stationary points occur where $f'(x) = 0$ giving

$6x^2 + 18x - 24 = 0$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4, x = 1$

| $x$ | $-10$ | $-4$ | 0 | 1 | 10 |
|---|---|---|---|---|---|
| $f'(x)$ | $+$ | 0 | $-$ | 0 | $+$ |
| *Shape* | | | | | |

So, the function is decreasing between $-4$ and 1, giving $-4 < x < 1$.

**2024 Paper 1Question 3, (2)**

$y = (5x^2 + 3)^7$

$\frac{dy}{dx} = 7(5x^2 + 3)^6 \times 10x$

$= 70x(5x^2 + 3)^6$

**2024 Paper 1 Question 12, (4)**

$f(x) = 12\sqrt[3]{x} = 12x^{\frac{1}{3}}$

$f'(x) = 4x^{-2/3} = \frac{4}{x^{\frac{2}{3}}}$

$f'(a) = \frac{4}{a^{\frac{2}{3}}}$

$\frac{4}{a^{\frac{2}{3}}} = 1$

$a^{\frac{2}{3}} = 4$

$\sqrt[3]{a^2} = 4$

$a^2 = 64s$

$a = 8$

**2024 Paper 2 Question 2, (5)**

$$y = \frac{8}{x^3} = 8x^{-3}$$

$$\frac{dy}{dx} = -24x^{-4} = \frac{-24}{x^4}$$

$$m = \frac{-24}{2^4} = -\frac{24}{16} = -\frac{3}{2}$$

When $x = 2, y = \frac{8}{2^3} = 1$

Using $y - b = m(x - a)$ with $(2, 1)$ gives

$$y - 1 = -\frac{3}{2}(x - 2)$$

$2y - 2 = -3(x - 2)$

$2y - 2 = -3x + 6$

$2y = -3x + 8$

$$y = -\frac{3}{2}x + 4$$

**2025 Paper 1 Question 1, (4)**

$y = x^3 - 2x^2 + 5$

When $x = 2, y = 2^3 - 2(2)^2 + 5 = 5$. So, we have the point $(2, 5)$

$$\frac{dy}{dx} = 3x^2 - 4x$$

$m = 3(2)^2 - 4(2) = 4$

Using $y - b = m(x - a)$ with $(2, 5)$ gives

$y - 5 = 4(x - 2)$

$y - 5 = 4x - 8$

$y = 4x - 3$

**2026 Paper 1 Question 7, (4)**

$y = 2x + 4\sqrt{x}$

$y = 2x + 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2 + 2x^{-\frac{1}{2}}$

$= 2 + \frac{2}{x^{\frac{1}{2}}}$

$= 2 + \frac{2}{\sqrt{x}}$

Substitute $x = 9$ to give the gradient, $m$

$m = 2 + \frac{2}{\sqrt{9}}$

$= 2 + \frac{2}{3}$

$= 2\frac{2}{3}$

**2026 Paper 2 Question 9, (4)**

$f(x) = 4x^3 - 12x^2 + 7$

$f'(x) = 12x^2 - 24x$

For strictly decreasing $f'(x) < 0$

$12x^2 - 24x < 0$

$12x(x - 2) < 0$

Roots are at $x = 0$ and $x = 2$

Consider a parabola with these roots; for decreasing identify the part of the graph under the $x$ axis, giving the range of values $0 < x < 2$