Higher Mathematics

Composite Functions – Solutions

Marks are indicated in brackets after each question

2013 Paper 1 Question 1, (2)

$$\begin{aligned} g(f(x)) = g(x^2+1) &= 3(x^2+1) - 4 \\ &= 3x^2 + 3 - 4 \\ &= 3x^2 - 1 \end{aligned}$$

2014 Paper 2 Question 3, (2)

a) $f(g(x)) = f(x+3) = (x+3)(x+3-1) + q$

$$= (x+3)(x+2) + q$$

$$= x^2 + 5x + 6 + q$$

b) $f(g(x)) = x^2 + 5x + 6 + q = x^2 + 5x + (6+q)$

$a = 1, b = 5, c = 6 + q$

For equal roots $b^2 - 4ac = 0$

$25 - 4 \cdot 1 \cdot (6+q) = 0$

$25 - 24 - 4q = 0$

$q = \frac{1}{4}$

2015 Paper 1 Question 5, (2) (1)

a) $g(x) = 6 - 2x$

$y = 6 - 2x$

Interchange x & y to give

$x = 6 - 2y$

Solve for y

$y = 3 - \frac{1}{2}x$

So, $g^{-1}(x) = 3 - \frac{1}{2}x$.

b) $g\left(g^{-1}(x)\right) = x$ since g and g^{-1} are inverses of each other.

2015 Paper 2 Question 2, (2) (3) (2)

a) $f\left(g(x)\right) = f((1+x)(3-x)+2)$

$= 10 + (1+x)(3-x) + 2$

Simplifying gives

$f\left(g(x)\right) = 15 + 2x - x^2$

b) $f\left(g(x)\right) = -x^2 + 2x + 15$

$= -(x^2 - 2x) + 15$

Completing the square of the inside of the bracket

$= -[(x-1)^2 - 1] + 15$

$= -(x-1)^2 + 16$

c) $h(x) = \frac{1}{f(g(x))} = \frac{1}{-(x-1)^2+16}$

Restrictions on the domain of $h(x)$ occur where $-(x-1)^2 + 16 = 0$

Let $-(x-1)^2 + 16 = 0$

$(x-1)^2 = 16$

$x - 1 = 4 \qquad x - 1 = -4$

$x = 5 \qquad x = -3$

So, $x = -3, x = 5$ cannot be in the domain of $h(x)$.

2016 Paper 1 Question 12, (2) (3)

a) $h(x) = f\left(g(x)\right) = f(3-x) = 2(3-x)^2 - 4(3-x) + 5 = 2x^2 - 8x + 11$

b) $h(x) = 2x^2 - 8x + 11 = 2(x^2 - 4x) + 11$

$= 2[(x-2)^2 - 4] + 11$

$= 2(x-2)^2 - 8 + 11$

$= 2(x-2)^2 + 3$

2017 Paper 1 Question 1, (1) (2)

a) $f(g(x)) = f(2\cos x) = 5 \cdot 2\cos x = 10\cos x$

$f(g(0)) = 10 \cdot \cos 0 = 10$

b) $g(f(x)) = g(5x) = 2\cos(5x)$

2018 Paper 2 Question 6, (2) (1)

a) i) $f(g(x)) = 3 + \cos(2x)$

ii) $g(f(x)) = 2(3 + \cos x)$

$= 6 + 2\cos x$

2019 Paper 1 Question 12, (2) (1)

a) $f(g(x)) = f(5 - x) = \frac{1}{\sqrt{5 - x}}$

b) $f(g(x))$ is undefined where $5 - x \leq 0$ since you cannot square root a negative and you cannot divide by zero.

So, $5 - x \leq 0$

$5 \leq x$

$x \geq 5$

2022 Paper 2 Question 5, (2) (1) (4)

a) i) $f(g(x)) = f(3x + 5)$

$= (3x + 5)^2 - 2$

$= 9x^2 + 15x + 15x + 25 - 2$

$= 9x^2 + 30x + 23$

ii) $g(f(x)) = g(x^2 - 2)$

$= 3(x^2 - 2) + 5$

$= 3x^2 - 1$

b) $f(g(x)) < g(f(x))$

$9x^2 + 30x + 23 < 3x^2 - 1$

$6x^2 + 30x + 24 < 0$

$x^2 + 5x + 4 < 0$

$(x + 4)(x + 1) < 0$

Consider the graph of $y = (x + 4)(x + 1)$ to see that $-4 < x < -1$

2023 Paper 1 Question 13, (1) (2) (1) (3)

a) i) $f(g(x)) = f(2x)$

$= 2\sin 2x$

$f\left(g\left(\frac{\pi}{6}\right)\right) = 2\sin 2(\frac{\pi}{6})$

$= 2\sin\frac{\pi}{3}$

$= 2\ x\ \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

ii) $f(g(x)) = f(2x)$

$= 2\sin 2x$

b) i) $f(p) = \frac{1}{3}$ and $f(p) = 2\sin p$

Equating gives $2\sin p = \frac{1}{3}$

$\sin p = \frac{1}{6}$

ii) $f(g(p)) = f(2p)$

$= 2\sin 2p$

$= 2(2\sin p\cos p)$

$= 2[\left(2\cdot\frac{1}{6}\right)\cos p]$

To determine the value of $\cos p$ use Pythagoras and SohCahToa based on $\sin p = \frac{1}{6}$.

In other words, based on opposite $= 1$ and hypotenuse $= 6$.

By Pythagoras, the adjacent side $= \sqrt{35}$.

$$f\big(g(p)\big) = 2[\left(2 \cdot \tfrac{1}{6}\right)\cos p] = 2[\left(2 \cdot \tfrac{1}{6}\right)\left(\tfrac{\sqrt{35}}{6}\right)]$$

$$= \frac{\sqrt{35}}{9}$$

2024 Paper 2 Question 8, (2) (2)

a) $f\big(g(x)\big) = f(x+1)$

$= 2(x+1)^2 - 18$

$= 2(x^2 + 2x + 1) - 18$

$= 2x^2 + 4x + 2 - 18$

$= 2x^2 + 4x - 16$

b) $\frac{1}{f(g(x))} = \frac{1}{2x^2 + 4x - 1}$

The function is undefined where the denominator, $2x^2 + 4x - 16$, equals zero.

$2x^2 + 4x - 16 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, x = 2$

2025 Paper 2 Question 12, (2) (2)

a) $h(x) = f\big(g(x)\big) = f(1 - x^3)$

$= (1 - x^3)^5 + 3$

b) $h'(x) = 5(1 - x^3)^4 \times -3x^2$

$= -15x^2(1 - x^3)^4$

2026 Paper 1 Question 5, (2) (1) (1)

a) i) $f(g(x)) = f(x+3)$

$= 2(x+3)^2 + 5$

$= 2(x^2 + 6x + 9) + 5$

$= 2x^2 + 12x + 23$

a) ii) $g(f(x)) = g(2x^2 + 5)$

$= 2x^2 + 5 + 3$

$= 2x^2 + 8$

b) The range of $g(f(x))$ is the range of the function $2x^2 + 8$. The range is the set of output values of the function which is equivalent to the range of y values on the graph of the function. The function $2x^2 + 8$ is a parabola with turning point at $(0, 8)$. So, the range of the function is the set $8 \leq y \leq \infty$