National 5 Mathematics

Using Trigonometry - Solutions

Marks are indicated in brackets after each question number

2014 Paper 1 Question 5, (3)

Using the Sine Rule gives

$$\frac{k}{\sin K} = \frac{l}{\sin L}$$

$$\frac{LM}{0.4} = \frac{18}{0.9}$$

$$LM = 0.4\ x\ \frac{18}{0.9}$$

$$LM = 0.4\ x\ 20$$

$$LM = 8$$

2014 Paper 2 Question 10, (3) (2)

a) Using the Cosine Rule, we have

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{11^2+8^2-13^2}{2x11x8}$$

$$= 0.09 \ldots.$$

$$B = cos^{-1}(0.09 \ldots)$$

$$= 85°.$$

b) Extending the line AB gives 'F' angles with the two North lines, with the 'F' angles being 60° & 120°.

So, shaded angle $= 360 - 85 - 120 = 155°$.

2015 Paper 2 Question 3, (3)

Using the Cosine Rule gives

$c^2 = a^2 + b^2 - 2ab\cos C$

$(AB)^2 = 1.35^2 + 1.2^2 - 2\ x\ 1.2\ x\ 1.35\ x\cos 35$

$(AB)^2 = 0.613\ldots$

$AB = 0.78$

So, $AB = 0.78\ km$.

2015 Paper 2 Question 13, (4)

$PQR = 180 - 128 = 56°$

$QRP = 180 - (52 + 72) = 56°$

Using the Sine Rule gives

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{q}{\sin 52°} = \frac{25}{\sin 56°}$$

$$q = \frac{25\sin 52°}{\sin 56°} = 23.67$$

So, the distance is 23.67 km.

2016 Paper 2 Question 8, (3)

Using the Sine Rule gives

$$\frac{\sin x}{150} = \frac{\sin 66}{140}$$

$$\sin x = \frac{\sin 66}{140}\ x\ 150$$

$\sin x = 0.978\ldots$

$x = sin^{-1}(0.978\ldots)$

$x = 78.2°$.

2016 Paper 2 Question 16, (4)

Using Pythagoras gives $DE = \sqrt{4^2 - 3^2} = \sqrt{7}$.

Using the Sine Rule on ADE gives

$$\frac{\sin A}{a} = \frac{\sin E}{e}$$

$$\frac{\sin A}{\sqrt{7}} = \frac{\sin 90}{4}$$

$$\sin A = \frac{\sqrt{7}\sin 90}{4}$$

$$= 0.661 \ldots$$

$$A = sin^{-1}(0.661 \ldots)$$

$$= 41°.$$

2017 Paper 2 Question 10, (4)

$EDF = 126 - 90 = 36°$

$DEF = 360 - 230 - 90 = 40°$

So, $DFE = 180 - 36 - 40 = 104°$.

Using the Sine Rule gives

$$\frac{f}{\sin F} = \frac{e}{\sin E}$$

$$\frac{15}{\sin 104} = \frac{DF}{\sin 40}$$

$$DF = \frac{15 \sin 40}{\sin 104}$$

$DF = 9.9$

Distance $= 9.9\ km$.

2018 Paper 1 Question 10, (3)

$$z^2 = x^2 + y^2 - 2xy\cos Z$$

$$= 8^2 + 10^2 - 2(8)(10)(\tfrac{1}{8})$$

$$= 164 - 20$$

$$= 144$$

$$z = 12$$

So, $XY = 12$ cm.

2018 Paper 2 Question 9, (3)

Using the Sine Rule gives

$$\frac{20}{\sin 37} = \frac{DC}{\sin 105}$$

$$DC = \frac{20 \sin 105}{\sin 37}$$

$$= 32 \text{ cm}$$

2018 Paper 2 Question 13, (4)

$$\cos T = \frac{5.6^2 + 10.3^2 - 7.2^2}{2 \ x \ 5.6 \ x \ 10.3}$$

$$= \frac{85.61}{115.36}$$

$$= 0.742 ..$$

$$T = cos^{-1}(0.742 ..)$$

$$= 42°$$

$$\text{Bearing} = 240 + 42$$

$$= 282°.$$

2019 Paper 2 Question 7, (3)

The smallest angle is at vertex Z.

Using the Cosine Rule gives

$$Cos \ Z = \frac{8.5^2 + 7.2^2 - 6.3^2}{2 \ (8.5) \ (7.2)}$$

$$= \frac{84.4}{122.4}$$

$$Z = cos^{-1}\left(\frac{84.4}{122.4}\right)$$

$$= 46.4°$$

2022 Paper 1 Question 9, (2)

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{3^2 + 7^2 - 5^2}{2(3)(7)}$$

$$\cos B = \frac{9 + 49 - 25}{42}$$

$$\cos B = \frac{9 + 49 - 25}{42}$$

$$\cos B = \frac{33}{42}$$

2022 Paper 2 Question 14, (5)

Start by making all of the angles in the triangle ACD.

Angle at $C = 180 - 28 = 152°$

Angle at $A = 180 - (152 + 12) = 16°$

Using the Sine Rule on triangle ACD gives

$$\frac{a}{\sin A} = \frac{d}{\sin D}$$

$$\frac{15}{\sin 16} = \frac{d}{\sin 12}$$

$$d = \frac{15 \sin 12}{\sin 16} = 11.3\ m$$

Next, make all of the angles in the triangle ABC.

Angle at $A = 180 - (90 + 28) = 62°$

Using the Sine Rule on triangle ABC gives

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 62} = \frac{11.3}{\sin 90}$$

$$a = \frac{11.3 \sin 62}{\sin 90}$$

$$a = 9.98\ m$$

2023 Paper 1 Question 6, (3)

Using the Cosine rule gives

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 6^2 + 5^2 - 2(6)(5)(\frac{1}{5})$$

$$c^2 = 36 + 25 - 12$$

$$c^2 = 49$$

$$c = 7\ m$$

2023 Paper 2 Question 4, (3)

Using the Sine Rule gives

$$\frac{\sin J}{j} = \frac{\sin K}{k}$$

$$\frac{\sin 25}{7} = \frac{\sin K}{10}$$

$$\sin K = \frac{10 \sin 25}{7}$$

$$\sin K = 0.60374 \ldots$$

$$K = sin^{-1}(0.60374 \ldots)$$

$$K = 37.1°$$

$$JKL = 37.1°$$

2023 Paper 2 Question 15, (4)

Use SOHCAHTOA on triangle ABC to give

$$\sin A = \frac{8}{18}$$

$$A = sin^{-1}\left(\frac{8}{18}\right) = 26.4°$$

Area of triangle ADE is given by

$$\text{Area} = \tfrac{1}{2} de \sin A$$

$$= \tfrac{1}{2} d \sin 26.4$$

Since the area of ADE is 160 we have

$$\frac{1}{2} d(24) \sin 26.4 = 160$$

Multiply both sides by 2 to give

$$24 \sin 26.4\, d = 320$$

Rearranging gives

$$d = \frac{320}{24 \sin 26.4}$$

$$d = 30\ cm$$

So, $AE = 30\ cm$

2024 Paper 2 Question 3, (3)

Using the Cosine rule:

$$Cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$Cos\ A = \frac{18^2 + 25^2 - 34^2}{2(18)(25)} = \frac{-207}{900}$$

$$A = cos^{-1}\left(\frac{-207}{900}\right) = 103.3°$$

2024 Paper2 Question 13, (5)

Consider the triangle ABC. Angle $B = 180 - 30 - 40 = 110°$

Set up the Sine Rule on triangle ABC to give:

$$\frac{c}{sin\ 30°} = \frac{22}{\sin 110°}$$

$$c = \frac{22 \sin 30°}{\sin 110°} = 11.7$$

Consider the triangle ABD. Angle $D = 90°$ and angle $B = 180 - 90 - 40 = 50°$

Set up the Sine Rule on triangle ABD to give:

$$\frac{a}{sin\ 40°} = \frac{11.7}{\sin 90°}$$

$$a = \frac{11.7 \sin 40°}{\sin 90°} = 7.5$$

So, $BD = 7.5\ cm$

2025 Paper 1 Question 8, (2)

$A = (120, 2)$

2025 Paper 2 Question 12, (4)

Angle $A = 131 - 90 = 41°$

Set up the Sine Rule to give

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{200}{\sin 41°} = \frac{250}{\sin C}$$

Rearrange to give

$$\sin C = \frac{250\ \sin 41°}{200}$$

$$C = sin^{-1}\left(\frac{250\ \sin 41°}{200}\right) = 55.1°$$

So, angle $B = 180 - 41 - 55.1 = 83.9°$

The bearing of C from B is $360 - 90 - 83.9 = 186.1°$