National 5 Mathematics

Trigonometry - Solutions

Marks are indicated in brackets after each question number

2014 Paper 1 Question 10, (2)

The graph has been stretched vertically by a factor of 3 so $a = 3$.

The graph has been moved to the right by $40°$ so $b = -40$.

2014 Paper 2 Question 12, (3)

$11\cos x° - 2 = 3$

$11\cos x° = 5$

$\cos x° = \dfrac{5}{11}$

$x = cos^{-1}\left(\dfrac{5}{11}\right) = 63°$

Second solution is $360 - 63 = 297°$.

2015 Paper 1 Question 6, (2)

Since the amplitude is 4, $a = 4$.

Since there are 3 copies of the Sine graph is $0 \leq x \leq 360°$, $b = 3$.

2015 Paper 1 Question 7, (1) (1) (1)

a) i) $a = -2$

   ii) $b = -4$

b) $x = -4$

2015 Paper 1 Question 9, (2)

$\cos 90° = 0$

$\cos 100° < 0$ from inspection of the graph.

$\cos 300° > 0$ from inspection of the graph.

So, $\cos 100° < \cos 90° < \cos 300°$.

2016 Paper 1 Question 11, (2)

$tan^2 x \cos^2 x$

Use $\tan x = \frac{\sin x}{\cos x}$ to give

$$\left(\frac{\sin x}{\cos x}\right)^2 \cos^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$$

$$= \sin^2 x$$

2016 Paper 2 Question 14, (3)

$2 \tan x + 5 = -4$

$2 \tan x = -9$

$\tan x = -4.5$

$x = tan^{-1}(-4.5)$

$x = 77°$

Using CAST we have

$x = 180 - 77 = 103°$

$x = 360 - 77 = 283°$

2017 Paper 2 Question 15, (1) (1) (4)

a) $h = 40 + 23 \cos x$

   When $x = 60°$

   $h = 40 + 23 \cos 60°$

   $\quad = 51.5 \ m.$

b) Minimum height occurs where $x = 180°$

   $h = 40 + 23 \cos 180°$

   $\quad = 17 \ m.$

c) Let $h = 61$ to give

   $61 = 40 + 23 \cos x$

   $\cos x = \frac{21}{23}$

$$x = cos^{-1}\left(\frac{21}{23}\right) = 24°$$
$$x = 360 - 24 = 336°$$

2018 Paper 1 Question 6, (2)

$y = 5 \cos 4x$

$a = 5, b = 4$

2018 Paper 1 Question 10, (3)

$$z^2 = x^2 + y^2 - 2xy \cos Z$$
$$= 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{8}\right)$$
$$= 164 - 20$$
$$= 144$$
$$z = 12$$

So, $XY = 12$ cm.

2018 Paper 1 Question 12, (1)

Sketch the graph of $y = \cos x$

Mark on a horizontal line through 0.5.

The line passes through the graph where $x = 60$.

From the symmetry of the graph, $\cos 240° = -0.5$.

Or use a CAST diagram.

2018 Paper 1 Question 18, (2)

$\sin x \cos x \tan x$

Substitute $\tan x = \frac{\sin x}{\cos x}$ to give

$\sin x \cos x \dfrac{\sin x}{\cos x}$

$= \sin x \sin x$

$= sin^2 x$

2018 Paper 2 Question 8, (3)

$7 \sin x + 2 = 3$

$\quad \sin x = \frac{1}{7}$

$x = sin^{-1}\left(\frac{1}{7}\right) = 8.2°$

From CAST diagram

$x = 180 - 8.2 = 171.8°$.

2018 Paper 2 Question 17, (5)

Area of Triangle $= \frac{1}{2}(38)(55) \sin 75$

$\qquad\qquad = 1009.39 \; cm^2$.

Area of Sector $= \frac{75}{360} \; x \; \pi \; x \; 60$

$\qquad\qquad = 39.27 \; cm^2$.

Shaded Area $= 1009.39 - 39.27$

$\qquad\qquad = 970.12 \; cm^2$.

2019 Paper 1 Question 13, (2)

$x - $ co-ordinate of $A = 180 - 45 = 135$.

$y - $ co-ordinate of $A = -1 \; x \; 3 = -3$.

Co-ordinates of $A = (135, -3)$.

2019 Paper 2 Question 7, (3)

The smallest angle is at vertex Z.

Using the Cosine Rule gives

$Cos \; Z = \dfrac{8.5^2 + 7.2^2 - 6.3^2}{2 \, (8.5) \, (7.2)}$

$\qquad = \dfrac{84.4}{122.4}$

$Z = cos^{-1}\left(\dfrac{84.4}{122.4}\right)$

$\quad = 46.4°$.

2019 Paper 2 Question 14, (3)

$5 \cos x + 2 = 1$

$\qquad 5 \cos x = -1$

$\qquad\quad \cos x = -\frac{1}{5}$

$\qquad\quad \cos x = -0.2$

$cos^{-1}(0.2) = 78°$

From CAST solutions lie in quadrants 2 & 3, giving

$x = 180 - 78 = 102°.$

$x = 180 + 78 = 258°.$

2019 Paper 2 Question 17, (2)

$(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$

$\qquad\qquad = sin^2x + 2 \sin x \cos x + cos^2x$

$\qquad\qquad = sin^2x + cos^2x + 2 \sin x \cos x$

$\qquad\qquad [sin^2x + cos^2x = 1 \text{ from Trig Identities}]$

$\qquad\qquad = 2 \sin x \cos x + 1$

2022 Paper 1 Question 8, (1) (1)

a) $a = 3$

b) $b = 8$

2022 Paper 2 Question 9, (3)

$3 \sin x + 4 = 6$

$3 \sin x = 2$

$\sin x = \frac{2}{3}$

$sin^{-1}\left(\frac{2}{3}\right) = 41.8°$

From CAST $x = 41.8°$ and $x = 180° - 41.8° = 138.2°$

2022 Paper 2 Question 13, (2)

$$\frac{\sin x + 2\cos x}{\cos x}$$

$$= \frac{\sin x}{\cos x} + \frac{2\cos x}{\cos x}$$

$$= \tan x + 2$$

2023 Paper 1 Question 11, (1)

Either use the CAST diagram or the symmetry of the Sine Graph to answer this question.

$\sin 330° = -\sin 30° = -0.5$

2023 Paper 1 Question 13, (1) (1)

a) $a = 210$

b) $b = 1$

2023 Paper 2 Question 11, (4)

$h = 20\cos x + 147$

Substitute $h = 150$ to give

$150 = 20\cos x + 147$

$20\cos x = 3$

$$\cos x = \frac{3}{20}$$

$$cos^{-1}\left(\frac{3}{20}\right) = 81.4°$$

From CAST $x = 81.4°$ and $x = 360 - 81.4 = 278.6°$

2023 Paper 2 Question 13, (2)

$$2sin^2x + 2cos^2x = 2(sin^2x + cos^2x)$$

$$= 2(1)$$

$$= 2$$

2024 Question 8, (1) (1)

a) $a = 7$ **b)** $b = 2$

2024 Paper 2 Question 11, (3)

$17 \sin x + 1 = 9$

$17 \sin x = 8$

$\sin x = \dfrac{8}{17}$

$sin^{-1}\left(\dfrac{8}{17}\right) = 28.07°$

Using CAST $x = 28.07°$ and $x = 180° - 28.07° = 151.93°$

2024 Paper 2 Question 16, (3)

$3\cos^2 x - 1$ (1)

$\cos^2 x + \sin^2 x = 1$

Rearrange to give $\cos^2 x = 1 - \sin^2 x$

Substitute into (1) to give

$3(1 - \sin^2 x) - 1 = 3 - 3\sin^2 x - 1$

$\qquad\qquad\qquad\quad = 2 - 3\sin^2 x$

2025 Paper 2 Question 14, (4)

$h = 10 - 8\cos x$

Substitute $h = 13$ to give

$13 = 10 - 8\cos x$

$3 = -8\cos x$

$\cos x = \dfrac{3}{-8}$

$cos^{-1}\left(\dfrac{3}{8}\right) = 67.98$

Use CAST to give $x = 180 - 67.9 = 112.02°$ and $x = 180 + 67.9 = 247.9°$

So, the two values of x are $112.02°$ and $247.9°$