



## National 5 Mathematics

### Quadratic Equations - Solutions

Marks are indicated in brackets after each question number

#### **2016 Paper 1 Question 6, (2)**

$$f(x) = 7x^2 + 5x - 1$$

$$a = 7, b = 5, c = -1$$

$$b^2 - 4ac = 25 - 4 \times 7 \times (-1) = 53$$

Since  $b^2 - 4ac > 0$  there are two roots.

#### **2016 Paper 1 Question 12, (1) (3) (3)**

**a)** Area of rectangle =  $(2x + 1)(x + 8)$

**b)** Area of triangle =  $\frac{1}{2}(3x)(2(x + 5))$   
 $= 3x(x + 5)$

Area of rectangle = area of triangle

$$(2x + 1)(x + 8) = 3x(x + 5)$$

$$2x^2 + 17x + 8 = 3x^2 + 15x$$

Simplifying gives  $x^2 - 2x - 8 = 0$

**c)**  $x^2 - 2x - 8 = 0$

$$(x - 4)(x + 2) = 0$$

$$x = -2, x = 4$$

Since  $x$  is a length it cannot be negative, so  $x = 4$ .

$$\text{Length} = 8 + 4 = 12 \text{ cm.}$$

$$\text{Breadth} = (2 \times 4) + 1 = 9 \text{ cm.}$$



**2017 Paper 2 Question 4, (3)**

$$2x^2 + 5x - 4 = 0$$

$$a = 2, b = 5, c = -4$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times (-4)}}{2 \times 2}$$

$$x = \frac{-5 \pm \sqrt{25 + 32}}{4}$$

$$x = \frac{-5 + \sqrt{57}}{4} = 0.6$$

$$x = \frac{-5 - \sqrt{57}}{4} = -3.1$$

**2018 Paper 1 Question 5, (2)**

$$x^2 - 11x + 24 = 0$$

$$(x - 8)(x - 3) = 0$$

$$x - 8 = 0 \text{ and } x - 3 = 0$$

$$x = 8, x = 3$$

**2018 Paper 1 Question 8, (2)**

$$f(x) = 2x^2 + 4x + 5$$

$$a = 2, b = 4, c = 5$$

$$b^2 - 4ac = 4^2 - 4(2)(5)$$

$$= 16 - 40$$

$$= -24$$

So, no real roots.

**2018 Paper 1 Question 19, (4)**

**b)**  $x^2 - 6x - 81 = 0$

$$(x - 3)^2 - 90 = 0$$

$$(x - 3)^2 = 90$$

$$x - 3 = \pm\sqrt{90}$$

$$x = 3 \pm 3\sqrt{10}$$



### 2019 Paper 1 Question 15, (1) (4)

a)  $h = 12t - 5t^2$

Substitute  $t = 2$  to give

$$h = (12 \times 2) - 5(2^2)$$

$$= 24 - 20$$

$$= 4$$

4 metres.

b) Substitute  $h = -17$  to give

$$-17 = 12t - 5t^2$$

$$5t^2 - 12t - 17 = 0$$

$$(5t - 17)(t + 1) = 0$$

$$5t - 17 = 0$$

$$t = \frac{17}{5}$$

$$t = 3.4$$

$$t + 1 = 0$$

$$t = -1$$

Since  $t$  represents *time* this solution can be discarded.

So, the ball will hit the sea after 3.4 seconds.

### 2019 Paper 2 Question 6, (3)

$$3x^2 + 9x - 2 = 0$$

$$a = 3, b = 9, c = -2$$

$$\begin{aligned} x &= \frac{-9 \pm \sqrt{9^2 - 4(3)(-2)}}{2(3)} = \frac{-9 \pm \sqrt{81 + 24}}{6} \\ &= \frac{-9 \pm \sqrt{105}}{6} \end{aligned}$$

$$x = \frac{-9 + \sqrt{105}}{6} = 0.2$$

$$x = \frac{-9 - \sqrt{105}}{6} = -3.2$$



**2022 Paper 1 Question 15, (1) (4)**

$$\begin{aligned}\text{a) Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2}(3)(x + 12) \\ &= \frac{3}{2}(x + 12)\end{aligned}$$

$$\begin{aligned}\text{b) Area of rectangle} &= \text{base} \times \text{height} \\ &= (8 - x) \times 6 \\ &= 6(8 - x)\end{aligned}$$

Area of triangle = Area of rectangle

$$\frac{3}{2}(x + 12) = 6(8 - x)$$

$$\frac{3}{2}(x + 12) = 48 - 6x$$

$$3(x + 12) = 96 - 12x$$

$$3x + 36 = 96 - 12x$$

$$15x = 60$$

$$x = 4$$

**2022 Paper 2 Question 7, (4)**

$$4x^2 + 2x - 7 = 0$$

$$a = 4, b = 2, c = -7$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{116}}{8}$$

$$x = \frac{-2 + \sqrt{116}}{8} \text{ and } x = \frac{-2 - \sqrt{116}}{8}$$

$x = 1.1$  and  $x = -1.6$  to 2 significant figures.



### 2023 Paper 1 Question 5, (2)

$$f(x) = 4x^2 + 6x - 1$$

$$a = 4, b = 6, c = -1$$

$$\begin{aligned} b^2 - 4ac &= 6^2 - 4(4)(-1) \\ &= 36 + 16 \\ &= 42 \end{aligned}$$

Since  $b^2 - 4ac > 0$  there are two real and distinct roots.

### 2023 Paper 2 Question 14, (2) (4)

$$\begin{aligned} \text{a) Volume} &= l \times b \times h \\ &= (x + 7)(x)(2) \\ &= 2x(x + 7) \\ &= 2x^2 + 14x \end{aligned}$$

But since the volume is 45 we have

$$2x^2 + 14x = 45$$

$$2x^2 + 14x - 45 = 0$$

$$\text{b) We need to solve } 2x^2 + 14x - 45 = 0$$

Since this doesn't factorise we have to use the quadratic formula

$$a = 2, b = 14, c = -45$$

$$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(-45)}}{2(2)}$$

$$x = \frac{-14 \pm \sqrt{556}}{4}$$

$$x = \frac{-14 + \sqrt{556}}{4} \text{ and } x = \frac{-14 - \sqrt{556}}{4}$$

$$x = 2.4 \text{ and } x = -9.4$$

But since  $x$  is a length it cannot be negative, so  $x = 2.4$ .

So, the breadth is 2.4 cm.



### 2024 Paper 2 Question 8, (4)

$$3x^2 + 8x + 1 = 0 \text{ so } a = 3, b = 8, c = 1$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 3 \times 1}}{2 \times 3}$$

$$x = \frac{-5 \pm \sqrt{52}}{6}$$

$$x = \frac{-5 + \sqrt{52}}{6} = 0.37 \text{ and } x = \frac{-5 - \sqrt{52}}{6} = -2.04$$

### 2025 Paper 1 Question 11, (2)

$$f(x) = 3x^2 + 2x + 1$$

$$a = 3, b = 2, c = 1$$

$$b^2 - 4ac = 2^2 - 4(3)(1)$$

$$= 4 - 12$$

$$= -8$$

Since  $b^2 - 4ac < 0$  there are no real roots.

### 2025 Paper 1 Question 15, (1) (2) (3)

$$\begin{aligned} \text{a) Area Rectangle} &= (2x + 3)(x + 1) \\ &= 2x^2 + 2x + 3x + 3 \\ &= 2x^2 + 5x + 3 \end{aligned}$$

$$\begin{aligned} \text{b) Area Square} &= (x + 3)(x + 3) \\ &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9 \end{aligned}$$

$$\text{Area Rectangle} = \text{Area Square}$$

$$2x^2 + 5x + 3 = x^2 + 6x + 9$$

Simplify to give

$$x^2 - x - 6 = 0$$



**c)**  $x^2 - x - 6 = 0$

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

Since  $x$  cannot be negative in this case we can dismiss  $x = -2$

So, the length is  $2(3) + 3 = 9 \text{ cm}$  and the length is  $3 + 1 = 4 \text{ cm}$