National 5 Mathematics

Properties of Shapes - Solutions

Marks are indicated in brackets after each question number

2014 Paper 1 Question 12, (4)

Consider the right-angled triangle PAC

$PC = 15$ since it is the radius

$AC = 27 - 15 = 12$ since $CB = 15$

Using Pythagoras' Theorem gives

$(PC)^2 = (PA)^2 + (AC)^2$

$15^2 = (PA)^2 + 12^2$

$15^2 - 12^2 = (PA)^2$

$81 = (PA)^2$

$PA = 9$

So, $PQ = 2\ x\ 9 = 18\ cm.$

2014 Paper 2 Question 13, (5)

Area of sector MON $= \frac{50}{360}\ x\ \pi\ x\ 7^2 = 21.4\ m^2$

Area of triangle MON $= \frac{1}{2}\ x\ 7\ x\ 7\ x \sin 50° = 18.8\ m^2$

Area of Chord at MN $= 2.6\ m^2$

Area of circle $= \pi\ x\ 7^2 = 153.9\ m^2$

Area of cross-section $= 153.9 - 2.6 = 151.3\ m^2$.

2015 Paper 1 Question 3, (3)

DFE $= 90°$ since triangle inscribed in a circle with one side being the diameter

So, FDE $= 180 - 90 - 64 = 26°$.

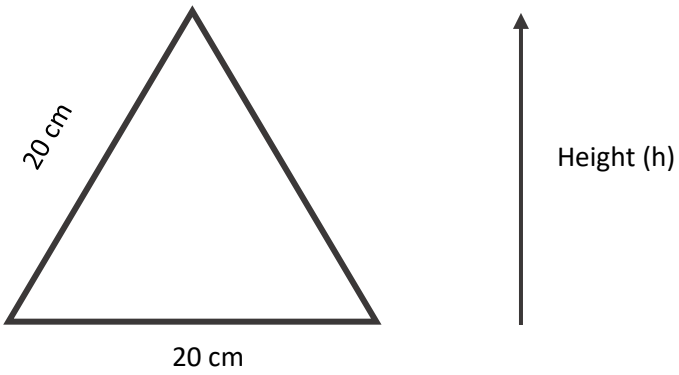
ABO $= 90°$ since tangent to the circle

So, OBD $= 90 - 77 = 13°$.

And BDO = 13° since isosceles triangle.

So, BDF = 26 + 13 = 39°.

2015 Paper 2 Question 11, (4)



Using Pythagoras to calculate height gives

$20^2 = 10^2 + h^2$

$400 = 100 + h^2$

$300 = h^2$

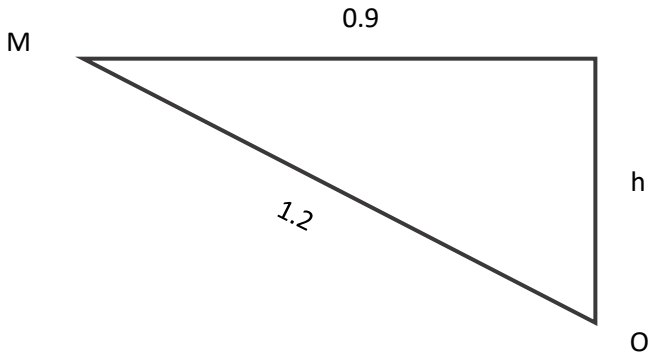
$h = \sqrt{300} = 17.32\ cm$

Area of triangle $= \frac{1}{2}\ x\ 17.32\ x\ 20 = 173.2\ cm^2$.

Area of table top $= 173.2\ x\ 6 = 1,039.2\ cm^2$.

2015 Paper 2 Question 12, (4)

Construct a right triangle from the midpoint of ML with O & M



Using Pythagoras gives

$1.2^2 = 0.9^2 + h^2$

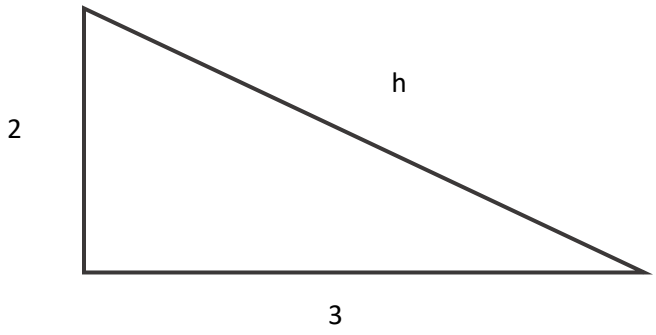
Solving gives $h = 0.79\ m$

So, depth of milk $= 0.79 + radius = 0.79 + 1.2 = 2.78\ m$

2016 Paper 1 Question 7, (1) (3)

a) $B = (8, 4, 0)$ by inspection of the graph.

b) Create a right-angled triangle in the base.



Using Pythagoras, we have

$h = \sqrt{2^2 + 3^2} = \sqrt{13}$

$(AV)^2 = 6^2 + (\sqrt{13})^2$

$= 49$

$AV = 7$

2016 Paper 2 Question 5, (3)

$EOA = 180 - 143 = 37°$.

So, $OAC = 37°$ since Z angle.

So, $CAB = 90 - 37 = 53°$ since tangent to circle makes 90° angle with radius.

$ACB = 53°$ since same angle as CAB.

So, $B = 180 - (53\ x\ 2) = 74°$.

2016 Paper 2 Question 15, (4)

Let M be the midpoint of AB.

$MB = 4.5$.

Make a right triangle OMB giving

$6.6^2 = (OM)^2 + 4.5^2$ by Pythagoras

$(OM)^2 = 6.6^2 - 4.5^2 = 23.31$

$OM = \sqrt{23.31} = 4.8$ cm.

Height = OM + radius

$= 4.8 + 6.6$

$= 11.4\ cm.$

2017 Paper 1 Question 9, (3)

OBE = 90°.

OBD = 90 − 58 = 32°.

ODB = 32° since isosceles triangle.

DOB = 180 − (32 x 2) = 116°.

BOC = 180 − 116 = 64°.

CAB = 180 − 90 − 64 = 26°.

2017 Paper 2 Question 3, (3)

Using the Cosine Rule gives

$p^2 = q^2 + r^2 - 2qr \cos P$

$= 180^2 + 150^2 - 2\ x\ 180\ x\ 150\ x \cos 147$

$= 170,380$

$p = \sqrt{170{,}380} = 413.$

Length $= 413\ m.$

2017 Paper 2 Question 13, (4)

Let C be the midpoint of AB.

Then, $AC = 24\ cm.$

Let D be the midpoint of AC.

Then, $AD = 12\ cm.$

Construct a right angled triangle A, C_1, D.

Using Pythagoras gives

$14^2 + 12^2 = (DC_1)^2$

$196 + 144 = (DC_1)^2$

$DC_1 = \sqrt{340} = 7.2\ cm$

Height $= (7.2\ x\ 2) + (14\ x\ 2) = 42.4\ cm.$

2018 Paper 1 Question 9, (4)

$360 \div 10 = 36$

$180 - 36 = 144$

$144 \div 2 = 72$

$180 - 72 = 108$

$17 + 108 = 125$

Shaded Area $= 180 - 125 = 55°$.

2019 Paper 1 Question 11, (3)

All of the angles at $O = 360 \div 5 = 72°$

$AOB = 72°$

$FOB = 180 - 72 = 108°$

$$OFB = \frac{180 - 108}{2}$$

$$= 36°$$

2019 Paper 2 Question 5, (2)

$A = (3, 0, 0)$ $B = (3, 3, 8)$

2019 Paper 2 Question 18, (4)

Create a right angled triangle TSB.

Since TS & SB are the radius of the circle they have length 7.5 cm.

By Pythagoras, $TB = \sqrt{7.5^2 + 7.5^2}$

$= 10.6\ cm$

TB is the radius of the larger circle, so TD also has length 10.6 cm.

So, height $= 10.6 + 15 = 25.6\ cm.$

2022 Paper 1 Question 4, (3)

$ACO = 90°$

$COE = 180° - 68° = 112°$

$OCE = \frac{180° - 112°}{2} = 34°$

$ACE = 90° + 34° = 124°$

2023 Paper 2 Question 5, (2)

Angles at the centre $= 360 \div 10 = 36°$

Other angles in each triangle $= \frac{180 - 36}{2} = 72°$

Using the straight line gives $180 - 72 - 72 = 36°$

Shaded area angle $= 90 + 36 = 126°$

2024 Paper 1 Question 10, (4)

$OBC = 90°$

$OFD = 180 - 125 = 55°$

So, $ODF = 55°$ and $FDE = 90 - 55 = 35°$

So, $DEF = 180 - 125 - 35 = 20°$

$BCD = 180 - 90 - 20 = 70°$

2025 Paper 2 Question 7, (2)

Let the centre point be O.

Angle at the centre $= 360 \div 5 = 72°$

$AEO = (180 - 72) \div 2 = 54°$

$FAE = 180 - 54 - 54 = 72°$

$FEA = 180 - 65 - 72 = 43°$