Differential Equations Formula List

1st Order Equations

For the 1st order linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ the integrating factor is given by $e^{\int p(x)\,dx}$.

2nd Order Homogeneous Equations

The homogeneous 2nd order linear differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ has characteristic equation $am^2 + bm + c = 0$.

The solutions of the equation are given by:

Where $b^2 - 4ac > 0$ with $m = \alpha, m = \beta, y = Ae^{\alpha x} + Be^{\beta x}$

Where $b^2 - 4ac = 0$ with $m = \alpha, y = (A + Bx)e^{\alpha x}$

Where $b^2 - 4ac < 0$ with $m = p \pm qi, y = A\,cos\,qx + B\,sin\,qx$

2nd Order Non-Homogeneous Equations

The homogeneous 2nd order linear differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ has characteristic equation $am^2 + bm + c = 0$.

The form of the particular integral is based on the $f(x)$ function.

Form of $f(x)$	Form of Particular Integral
k	$y = \lambda$
kx	$y = \lambda + \mu x$
kx^2	$y = \lambda + \mu x + \nu x^2$
ke^{px}	$y = \lambda e^{3x}$

Euler's Method

1st Order $y_{n+1} \approx y_n + h(\frac{dy}{dx})_n$

2nd Order $y_{r+1} \approx 2y_r - y_{r-1} + h^2(\frac{dy}{dx})_r$

Series Solutions Using Taylor Series

The following solutions use the Taylor Series expansions:

$$C: y = y_0 + (x - x_0)\frac{dy}{dx}\bigg|_{x_0} + \frac{(x - x_0)^2}{2!}\frac{d^2y}{dx^2}\bigg|_{x_0} + \frac{(x - x_0)^3}{3!}\frac{d^3y}{dx^3}\bigg|_{x_0} + \ldots$$

$$D: y = y_0 + x\frac{dy}{dx}\bigg|_0 + \frac{x^2}{2!}\frac{d^2y}{dx^2}\bigg|_0 + \frac{x^3}{3!}\frac{d^3y}{dx^3}\bigg|_0 + \ldots$$