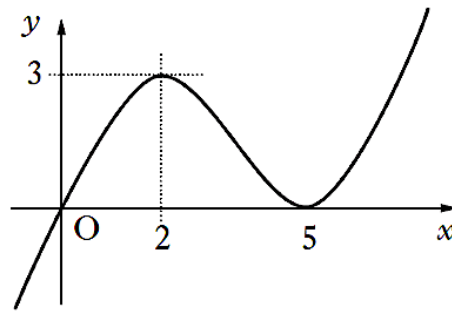


Higher Maths – Practice Paper – Non-Calculator

- 1) Find the equation of the line which passes through the point $(-2, -1)$ and is parallel to the line with equation $5x + 3y - 6 = 0$. (3)
- 2) Evaluate $\log_5 250 - \frac{1}{3}\log_5 8$. (3)
- 3) A function $g(x)$ is defined by $g(x) = \frac{1}{5}x - 4$ (3)
Find the inverse function, $g^{-1}(x)$.
- 4) Differentiate $y = \sqrt{x^3} - 2x^{-1}$. (3)
- 5) Find the equation of the line which passes through the point $(3, 0)$ and makes an angle of 45° with the positive direction of the x-axis. (3)
- 6) Evaluate $\int_1^6 \frac{2}{(x+3)^{-\frac{1}{2}}} dx$. (4)
- 7) a) Show that $\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x = \sin^3 x$. (3)
b) Hence, differentiate $\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x$. (3)
- 8) Solve for x the equation $\log_7(x^2 + 1) - 2 \log_7 x = \log_7 5$, where $x > 0$ (4)
- 9) Solve the equation $\cos 2x = 5 \cos x - 3$ for $0 \leq x < 360^\circ$. (5)
- 10) Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. (3)
- 11) Given that $f(x) = 4 \sin\left(3x - \frac{\pi}{3}\right)$, evaluate $f'\left(\frac{\pi}{6}\right)$. (3)

12) The diagram shows part of the graph of $y = f(x)$.

(3)



What is the equation of this curve?

13) For the polynomial $6x^3 + 7x^2 + ax + b$ it is known that $(x + 1)$ is a factor and 72 is the remainder when it is divided by $(x - 2)$

a) Determine the values of a and b . (4)

b) Hence, fully factorise the polynomial. (3)

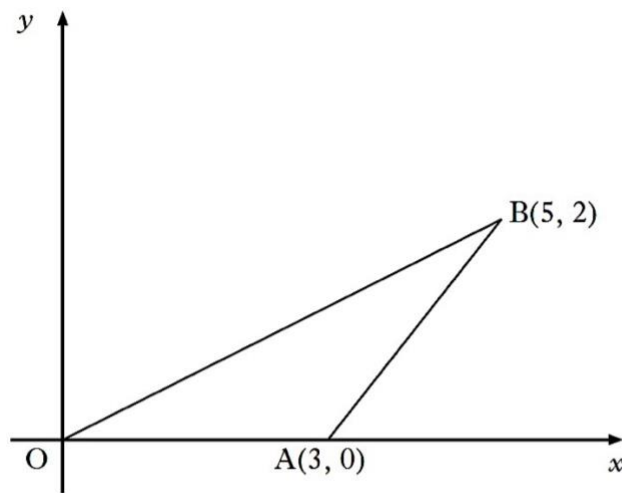
14) The point $A(8, -5)$ lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$ (4)

Find the equation of the tangent to the circle at point A .

15) Determine whether the line with equation $y = 3x$ is a tangent or chord to the circle with equation $(x - 2)^2 + (y - 1)^2 = 25$. (4)

Higher Maths – Practice Paper – Calculator

1) $A(3, 0)$, $B(5, 2)$ and the origin are the vertices of a triangle as shown in the diagram.



a) Obtain the equation of the perpendicular bisector of AB . (4)

b) The median from A has equation $y + 2x = 6$.

Find T , the point of intersection of this median and the line AB . (2)

c) Calculate the angle that AT makes with the positive direction of the x -axis. (2)

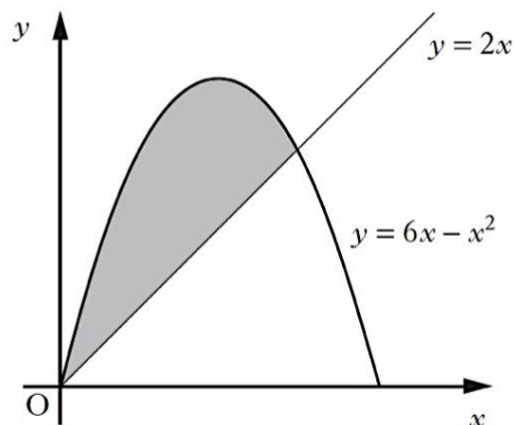
2) The equation $x^2 + (m - 3)x + m = 0$ has two real and distinct roots. (4)

Determine the range of values for m .

3) a) Express $2 \sin x + 2 \cos x$ in the form $k \cos(x - a)$. (4)

b) Hence solve the equation $2 \sin x + 2 \cos x = 2.81$ for $0 \leq x \leq 360^\circ$ (3)

4) Calculate the shaded area in the below diagram. (5)



5) Functions f and g are given by $f(x) = x^2 - 2$ and $g(x) = 3x + 5$. Find expressions for:

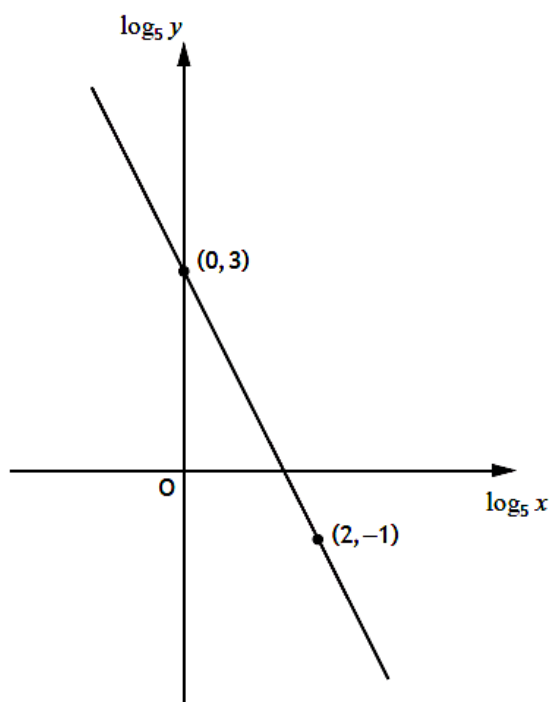
a) $f(g(x))$ (2)

b) $g(f(x))$ (1)

6) Given that $\int_4^t (3x + 4)^{-\frac{1}{2}} dx = 2$, find the value of t . (5)

7) Two variables, x and y , are connected by the equation $y = kx^n$.

The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown. Find the values of k and n . (5)



8) A sector with a particular fixed area has radius x cm. (6)

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}$$

Find the minimum value of P .

9) For the function, f , defined on a suitable domain, it is known that: (4)

$$f'(x) = \frac{2x+1}{\sqrt{x}} \text{ and } f(9) = 40$$

Express $f(x)$ in terms of x .

10) The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

P_0 is the initial concentration

P_t is the concentration after time t

t is the time, in days, after the application of the pesticide

a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.

(4)

b) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?

(3)

Higher Maths – Practice Paper – Non-Calculator – Solutions

1) $y = -\frac{5}{3}x - \frac{13}{3}$

2) 3

3) $g^{-1}(x) = 5x + 20$

4) $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + \frac{2}{x^2}$

5) $y = x - 3$

6) $\frac{76}{3}$

7) a) Proof using identities.

b) $3\sin^2 x \cos x$

8) $x = \frac{1}{2}$

9) $x = 60^\circ, x = 300^\circ$

10) $2(x + 3)^2 - 17$

11) $6\sqrt{3}$

12) $y = \frac{1}{6}x(x - 5)^2$

13) a) $a = -1$

b) $b = -2$

14) $y = \frac{1}{4}x - 7$

15) Chord. Points of intersection are $(-1, -3)$ and $(2, 6)$.

Or use the discriminant to show two solutions (roots).

Higher Maths - Practice Paper - Calculator - Solutions

1) a) $y = -x + 5$

b) $T = (1, 4)$

c) 63.4°

2) $m < 1, m > 9$

3) a) $2 \sin x + 2 \cos x = \sqrt{8} \cos(x - 45)^\circ$

b) $x = 51.5^\circ$

4) $\frac{32}{3}$ square units

5) a) $f(g(x)) = 9x^2 + 30x + 23$

b) $g(f(x)) = 3x^2 - 1$

6) $t = 15$

7) $k = 125, n = -2$

8) $x = 8$ giving $P = 32$ cm

9) $f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2$

10) a) $k = 0.028$ to 2 significant figures.

b) $p_t = 0.11 p_0$ so the concentration has decreased by 89%.