Trigonometry & Logarithms

Solutions

Radians & CAST

Q1) a) $180° = \pi$ radians

b) $360° = 2\pi$ radians

c) $90° = \frac{\pi}{2}$ radians

d) $30° = \frac{\pi}{6}$ radians

e) $45° = \frac{\pi}{4}$ radians

f) $60° = \frac{\pi}{3}$ radians

Q2) a) $10° = 10\ x\ \frac{\pi}{180}$
$= 0.17$

b) $40° = 40\ x\ \frac{\pi}{180}$
$= 0.7$

c) $305° = 305\ x\ \frac{\pi}{180}$
$= 5.3$

Q3) a) $\frac{3\pi}{2} = 270°$

b) $\frac{3\pi}{4} = 135°$

c) $2\pi = 360°$

d) $2.5 \text{ rad} = 2.5\ x\ \frac{180}{\pi} = 143°$

e) $1.5 \text{ rad} = 1.5\ x\ \frac{180}{\pi} = 86°$

f) $3.5 \text{ rad} = 3.5\ x\ \frac{180}{\pi} = 201°$

Q4) a) Sine is positive in quadrant 2

b) Cosine is positive in quadrant 1

c) Tangent is negative in quadrant 2

d) Cosine is positive in quadrant 1

e) Sine is negative in quadrant 4

f) Cosine is equal to 0 at 90° - this is best seen on the graph of $y = Cos\ x$

Q5) a) Sine is positive in quadrant 1

b) Cosine is negative in quadrant 2

c) Tangent is negative in quadrant 2

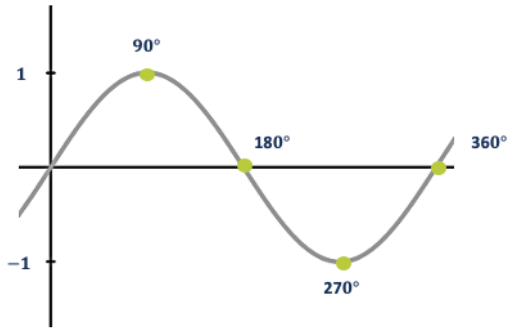
d) Cosine is positive in quadrant 1

e) Sine is negative in quadrant 4

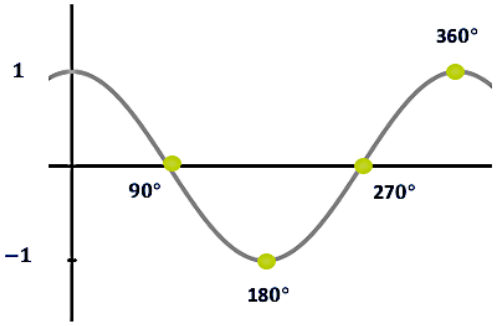
f) Cosine is positive in quadrant 1

Graphs of The Trigonometric Functions

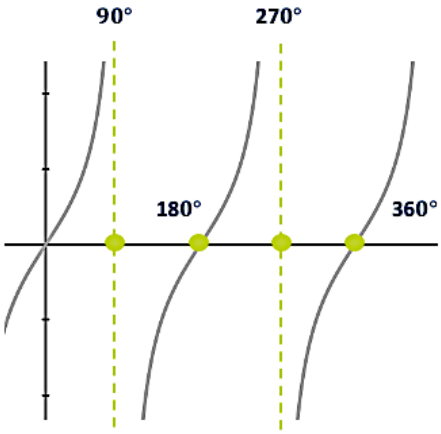
Q1) a) $y = \sin x$



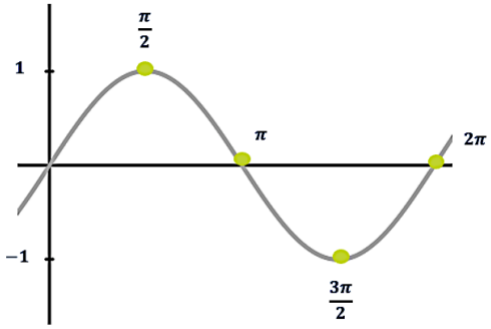
b) $y = \cos x$



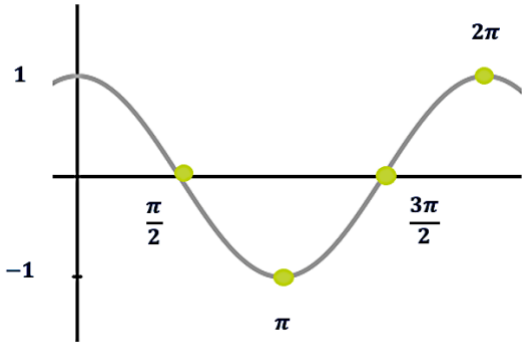
c) $y = \tan x$



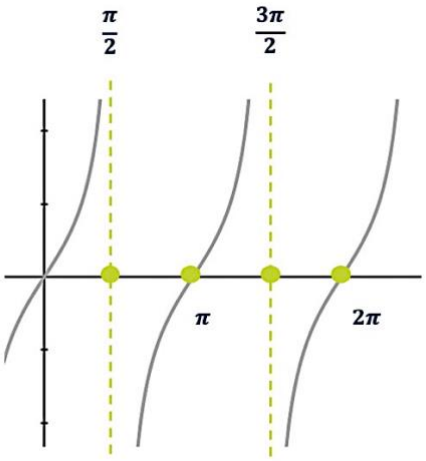
Q2) a) $y = \sin x$



b) $y = \cos x$



c) $y = \tan x$



Q3) a) Negative
b) Negative
c) Positive
d) Negative
e) Negative
f) Positive

Trigonometric Identities

Q1) a) Using Pythagoras the missing side of the triangle is 4.

$Sin\ 2a = 2\ Sin\ a\ Cos\ a$

Using SohCahToa, $Sin\ a = \frac{3}{5}$ and $Cos\ a = \frac{4}{5}$

$Sin\ 2a = 2\ x\ \frac{3}{5}\ x\ \frac{4}{5}$

$= \frac{24}{25}$

b) $Cos\ 2a = 2\ Cos^2 a - 1$

$= 2\ x\ (\frac{4}{5})^2 - 1$

$= \left(2\ x\ \frac{16}{25}\right) - 1$

$= \frac{32}{25} - \frac{25}{25}$

$= \frac{7}{25}$

Q2) $Sin\ 2x = 2\ Sin\ x\ Cos\ x$

From SohCahToa, $Sin\ x = \frac{1}{\sqrt{5}}$ and $Cos\ x = \frac{2}{\sqrt{5}}$

$$Sin\ 2x = 2\ x\ \frac{1}{\sqrt{5}}\ x\ \frac{2}{\sqrt{5}}$$

$$= \frac{4}{5}$$

Q3) a) $Sin\ (30° + 60°) = Sin\ 30°\ Cos\ 60° + Cos\ 30°\ Sin\ 60°$

$$= \left(\frac{1}{2}\ x\ \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\ x\ \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

b) $Cos\ (45° + 45°) = Cos\ 45°\ Cos\ 45° - Sin\ 45°\ Sin\ 45°$

$$= \left(\frac{1}{\sqrt{2}}\ x\ \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\ x\ \frac{1}{\sqrt{2}}\right)$$

$$= 0$$

c) $Cos\ (45° + 30°) = Cos\ 45°\ Cos\ 30° - Sin\ 45°\ Sin\ 30°$

$$= \left(\frac{1}{\sqrt{2}}\ x\ \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\ x\ \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

d) $Cos\ (30° - 45°) = Cos\ 30°\ Cos\ 45° + Sin\ 30°\ Sin\ 45°$

$$= \left(\frac{\sqrt{3}}{2}\ x\ \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\ x\ \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q2) a) $Sin\ \left(\frac{\pi}{3} + \frac{\pi}{6}\right) = Sin\ \frac{\pi}{3}\ Cos\ \frac{\pi}{6} - Cos\ \frac{\pi}{3}\ Sin\ \frac{\pi}{6}$

$$= \left(\frac{\sqrt{3}}{2}\ x\ \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\ x\ \frac{1}{2}\right)$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4} = \frac{1}{2}$$

b) $Cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right) = Cos\ \frac{\pi}{2}\ Cos\ \frac{\pi}{3} - Sin\ \frac{\pi}{2}\ Sin\ \frac{\pi}{3}$

$= \left(0\ x\ \frac{1}{2}\right) - \left(1\ x\ \frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{3}}{2}$

c) $Cos\left(\frac{\pi}{2}+\pi\right) = Cos\ \frac{\pi}{2}\ Cos\ \pi - Sin\ \frac{\pi}{2}\ Sin\ \pi$

$= (0\ x\ -1) - (1\ x\ 0) = 0$

d) $Cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right) = Cos\ \frac{\pi}{2}\ Cos\ \frac{\pi}{6} - Sin\ \frac{\pi}{2}\ Sin\ \frac{\pi}{6}$

$= \left(0\ x\ \frac{\sqrt{3}}{2}\right) - \left(1\ x\ \frac{1}{2}\right) = -\frac{1}{2}$

Exponentials & Logarithms

Q1) a) $\log_2 128 = 7$

b) $\log_5 125 = 3$

c) $\log_3 27 = 3$

d) $\log_4 64 = 3$

e) $\log_4 p = t$

f) $\log_g f = h$

Q2) a) $2^3 = 8$

b) $3^5 = 243$

c) $5^y = 4$

d) $4^2 = 16$

e) $x^0 = 1$

f) $5^4 = 625$

Q3) a) $\log_2 8 + 3\log_2 4 - \log_2 16$

$= 3 + (3\ x\ 2) - 4$

$= 5$

b) $3\log_{10} 5 + \log_{10} 8$

$= \log_{10} 5^3 + \log_{10} 8$

$= \log_{10} 125 + \log_{10} 8$

$= \log_{10} 1000$

$= 3$

c) $\log_4 36 - 2\log_4 12$

$= \log_4 36 - \log_4 12^2$

$= \log_4 36 - \log_4 144$

$= \log_4(\frac{36}{144}) = \log_4(\frac{1}{4}) = -1$

d) $2\log_3 9 + \log_3 5 - \log_3 15$

$= \log_3 9^2 + \log_3 5 - \log_3 15$

$= \log_3 81 + \log_3 5 - \log_3 15$

$= \log_3 27 = 3$

Q4) a) $\frac{1}{3}\log_4 64 = \log_4 64^{\frac{1}{3}}$

$= \log_4 4$

$= 1$

b) $\log_2 \frac{1}{8} = \log_2 1 - \log_2 8$

$= 0 - 3$

$= -3$

c) $\log_a a^p = p\log_a a$

$= p$

d) $\log_4 p^0 = 0\ x\ \log_4 p$

$= 0$