Series

Solutions

Arithmetic Sequences & Series

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Sums to Infinity

Sigma Notation

Arithmetic Sequences & Series

Q1) a) $u_n = 5n + 2$

i) $u_1 = 5(1) + 2 = 7$

$u_2 = 5(2) + 2 = 12$

$u_3 = 5(3) + 2 = 17$

$u_4 = 5(4) + 2 = 22$

ii) $a = 7$ and $d = 5$

b) $u_n = 9 - 2n$

i) $u_1 = 9 - 2(1) = 7$

$u_2 = 9 - 2(2) = 5$

$u_3 = 9 - 2(3) = 3$

$u_4 = 9 - 2(4) = 1$

ii) $a = 7$ and $d = -2$

c) $u_n = n - 10$

i) $u_1 = (1) - 10 = -9$

$u_2 = (2) - 10 = -8$

$u_3 = (3) - 10 = -7$

$u_4 = (4) - 10 = -6$

ii) $a = -9$ and $d = 1$

Q2) $x_n = 4n - 2$

Writing out the first 5 terms of the sequence gives $2, 6, 10, 14, 18, ..$

$d = 4$

Let $n = 15$ to give $a_{15} = 2 + (15 - 1)4$

$= 2 + 56$

$= 58$

Let $n = 20$ to give $a_{20} = 2 + (20 - 1)4$

$= 2 + 76$

$= 78$

Q3) $x_n = 3n + 5$

Writing out the first 5 terms of the sequence gives $8, 11, 14, 17, 20, ..$

$d = 3$

Let $n = 5$ to give $a_5 = 8 + (5 - 1)3$

$= 8 + 12$

$= 20$

Or we can just read a_5 from the list of terms above

Let $n = 8$ to give $a_8 = 8 + (8 - 1)3$

$= 8 + 27$

$= 35$

Q4) $x, 3x, 5x, \dots, 35x$

$a = x$ and $d = 2x$

Then $u_n = x + (n - 1)2x$

Set this equal to the final term in the sequence:

$x + 2nx - 2x = 35x$

$2nx = 36x$

$2n = 36$

$n = 18$

So, there are 18 terms in the sequence.

Q5) $a = 14$ and $u_4 = 32$

$u_n = 14 + (n-1)d$

Set this equal to $u_4 = 32$:

$14 + \big((4) - 1\big)d = 32$

$14 + 3d = 32$

$3d = 18$

$d = 6$

Q6) a) $S_{20} = \frac{20}{2}(2(3) + (20-1)4)$

$= 10(6 + (19)4)$

$= 10(82)$

$= 820$

b) $S_{14} = \frac{14}{2}(2(5) + (14-1)(-4))$

$= 7(10\text{-}52)$

$= -294$

Q7) $64 + 62 + 60 + \cdots = 0$

$a = 64, d = -2, S_n = 0$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$\frac{n}{2}\big(2(64) + (n-1)(-2)\big) = 0$

$\frac{n}{2}(128 - 2n + 2) = 0$

$\frac{n}{2}(130 - 2n = 0$

$n(65 - n) = 0$

$n = 0 \; or \; 65$

So, the number of terms is 65

Q8) $S = 2 + 4 + 6 + 8 + \cdots (50 \; terms)$

$a = 2, d = 2, n = 50$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S = \frac{50}{2}(2(2) + (50-1)2) = 2550$

Geometric Sequences & Series

Q1) a) $5, 15, 45, ..$

$a = 5$ and $r = 3$

$u_4 = 5(3)^3 = 135$

$u_5 = 5(3)^4 = 405$

$u_6 = 5(3)^5 = 1215$

b) $1, p, p^2, ..$

$a = 1\ and\ r = p$

$u_4 = 1(p)^3 = p^3$

$u_5 = 1(p)^4 = p^4$

$u_6 = 1(p)^5 = p^5$

Q2) $3, 6, 12, 24, 48, ..$

$a = 3, r = 2$

Let $n = 8$, to give $a_8 = 3\ x\ 2^7$

$= 3\ x\ 128$

$= 384$

Let $n = 12$, to give $a_{12} = 3\ x\ 2^{11}$

$= 3\ x\ 2{,}048$

$= 6, 144$

Q3) a) $2, 6, 18, 54, ..$

$a = 2, r = 3$

Let $n = 6$, to give $a_6 = 2\ x\ 3^5$

$= 2\ x\ 243$

$= 486$

$2, 6, 18, 54, ..$

$a = 2, r = 3$

Let $n = 8$, to give $a_6 = 2\ x\ 3^7$

$= 2\ x\ 2{,}187$

$= 4{,}374$

b) $1, -2, 4, -8, ..$

$a = 1, r = -2$

Let $n = 6$, to give $a_6 = 1\ x\ (-2)^5$

$= 1\ x - 32$

$= -32$

$1, -2, 4, -8, ..$

$a = 2, r = 3$

Let $n = 8$, to give $a_6 = 1\ x\ (-2)^7$

$= 1\ x - 128$

$= -128$

Q4) $ar^5 = 32 \qquad (1)$

$ar^2 = 4 \qquad (2)$

$\frac{ar^5}{ar^2} = \frac{32}{4} = 8 \Longrightarrow r^3 = 8 \Longrightarrow r = 2$

Substituting $r = 2$ into (2) gives

$a(2)^2 = 4 \Longrightarrow a = 1$

Q5) a) $1 + 2 + 4 + 8 + \cdots (8\ terms)$

$a = 1, r = 2\ and\ n = 8$

$S_8 = \frac{1(2^8 - 1)}{2 - 1}$

$= (2^8 - 1)$

$= 256 - 1$

$= 255$

b) $729 - 243 + 81 - \cdots - \frac{1}{3}$

$a = 729, r = -\frac{1}{3}$

The nth term is $-\frac{1}{3}$ and the nth term is given by ar^{n-1}

So, $-\frac{1}{3} = 729\left(-\frac{1}{3}\right)^{n-1}$

$-\frac{1}{2187} = \left(-\frac{1}{3}\right)^{n-1}$

$\left(-\frac{1}{3}\right)^7 = \left(-\frac{1}{3}\right)^{n-1}$

So, $n - 1 = 7 \Longrightarrow n = 8$. There are 8 terms in the series

$S_8 = \frac{729(1 - \left(-\frac{1}{3}\right)^8)}{1 - (-\frac{1}{3})}$

$= \frac{1640}{3} = 546\frac{2}{3}$

Q6) $a = 5, r = \frac{2}{3}$ $and\ n = 8$

$$S_8 = \frac{5(1 - \left(\frac{2}{3}\right)^8)}{1 - \frac{2}{3}}$$

$$= 14.4147$$

Q7) $a = a$ and $r = \sqrt{3}$

$$S_{10} = \frac{a(\sqrt{3}^{10} - 1)}{\sqrt{3} - 1}$$

$$= \frac{a(243 - 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{242a(\sqrt{3} + 1)}{3 - 1}$$

$$= 121a(\sqrt{3} + 1)$$

Sums to Infinity

Q1) a) $a = 10\ and\ S_\infty = 30$

$$\frac{10}{1 - r} = 30$$

$$10 = 30(1 - r)$$

$$10 = 30 - 30r$$

$$30r = 20$$

$$r = \frac{2}{3}$$

b) $a = -5\ and\ S_\infty = -3$

$$\frac{-5}{1 - r} = -3$$

$$-5 = -3(1 - r)$$

$$3r = -2$$

$$r = -\frac{2}{3}$$

Q2) $S_\infty = 60\ and\ r = \frac{2}{3}$

$$\frac{a}{1 - \frac{2}{3}} = 60$$

$$\frac{a}{\frac{1}{3}} = 60 = 20$$

Q3) $S_3 = 9 \; and \; S_\infty = 8$

$S_3 = \frac{a(1-r^3)}{1-r} = 9 \qquad (1)$

$S_\infty = \frac{a}{1-r} = 8 \qquad (2)$

Substituting (2) into (1) gives

$8(1-r^3) = 9$

$1 - r^3 = \frac{9}{8}$

$r^3 = -\frac{1}{8}$

$r = -\frac{1}{2}$

So from (2), $a = 8(1 + \frac{1}{2})$

$a = 12$

Sigma Notation

Q1) a) $\sum_{r=1}^{5}(3r+1) = 4 + 7 + 10 + 13 + 16 = 50$

$S_5 = 50$

b) $\sum_{r=1}^{6} 3r^2 = 3 + 12 + 27 + 48 + 75 + 108 = 271$

$S_6 = 273$

Q2) a) $2 + 4 + 6 + 8 = \sum_{r=1}^{4} 2r$

$S_4 = 20$

b) $2 + 6 + 18 + 54 + 162 = \sum_{r=1}^{5}(2(3)^{r-1})$

$S_5 = 242$

Q3) $\sum_{r=9}^{30}\left(5r - \frac{1}{2}\right) = 44\frac{1}{2} + \cdots + 149\frac{1}{2}$

$a = 44\frac{1}{2}, \; l = 149\frac{1}{2}, \; n = 22$

$S_{22} = \frac{22}{2}\left(44\frac{1}{2} + 149\frac{1}{2}\right)$

$S_{22} = 11(194)$

$S_{22} = 2134$