Functions & Polynomials

Solutions

Types of Function

Maximum & Minimum Values

Domain of a Function

Average Rate of Change

Polynomials

Types of Function

Q1) a) Linear

b) Quadratic

c) Trigonometric

d) Cubic

e) Cubic

f) Trigonometric

Maximum & Minimum Values

Q1) a) No maximum or minimum as straight lines continue indefinitely.

b) No maximum or minimum as straight lines continue indefinitely.

c) Minimum value is 0. There is no maximum value.

d) Minimum value is 3. There is no maximum value.

e) No maximum or minimum as straight lines continue indefinitely.

f) No maximum or minimum as straight lines continue indefinitely.

Domain of a Function

Q1) a) Restriction at $x = 6$

b) Restriction at $x = -9$

c) Restrictions are all $x > 12$

d) Restrictions are all $x < 8$

e) Let $(x + 2)(x - 6) = 0$

Giving $x = -2, x = 6$

So, restrictions occur at $x = -2, x = 6$

f) Let $x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 1, x = 2$

So, restrictions at $x = 1, x = 2$

Also, for $\sqrt{x}$, $x \geq 0$

So, restrictions are all $x < 0, x = 1, x$

Q2) a) Let $x^2 - 2x - 15 \geq 0$

$(x - 5)(x + 3) \geq 0$

$5 \leq x \leq -3$

Check elsewhere to learn how to solve quadratic inequations

So, the domain of f is all x; $5 \leq x \leq -3$

b) Let $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3, x = 4$

So, the domain of g is all x; $x \neq 3, x \neq 4$

c) Let $x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = -8, x = 2$

So, the domain of f is all x; $x \neq -8, x \neq 2$

d) Let $3x - 2x^2 = 0$

$x(3 - 2x) = 0$

$x = 0, x = \frac{3}{2}$

So, the domain of h is all x; $x \neq 0, x \neq \frac{3}{2}$

Average Rate of Change

Q1) a) $f(x) = 2x + 3$

$f(5) = 13$

$f(1) = 5$

Rate of change $= \frac{13-5}{5-1} = \frac{8}{4} = 2$

b) $f(x) = -2x + 5$

$f(3) = -1$

$f(-3) = 11$

Rate of change $= \frac{-1-11}{3-(-3)} = \frac{-12}{6} = -2$

c) $f(x) = x^2 - 2$

$f(-2) = 2$

$f(-6) = 34$

Rate of change $= \frac{2-34}{-2-(-6)} = \frac{-32}{4} = -8$

d) $f(x) = x^3$

$f(4) = 64$

$f(1) = 1$

Rate of change $= \frac{64-1}{4-1} = \frac{63}{3} = 21$

Polynomials

Q1) a)

3	1	-8	19	-12
		3	-15	12
	1	-5	4	0

Since the remainder is 0, $(x - 3)$ is a factor of $f(x)$.

b) $f(x) = (x-3)(x^2-5x+4)$

$= (x-3)(x-4)(x-1)$

Q2) a)

1	3	4	-5	-2
		3	7	2
	3	7	2	0

Since the remainder is 0, $(x-1)$ is a factor of $f(x)$.

b) $f(x) = (x-1)(3x^2+7x+2)$

$= (x-1)(3x+1)(x+2)$

Q3) a)

3	1	-2	-9	18
		3	3	-18
	1	1	-6	0

Since the remainder is 0, $(x-3)$ is a factor of $f(x)$.

b)

-2	1	5	2	-8	0
		-2	-6	8	0
	1	3	-4	0	0

Since the remainder is 0, $(x+2)$ is a factor of $f(x)$.

Q4) a) $(x-2)(x-1)(x+4)=0$

$x-2=0, x-1=0, x+4=0$

$x=2, x=1, x=-4$

b) $(x-3)(x-3)(x+2)=0$

$x-3=0, x+2=0$

$x=3, x=-2$

c) $(2x-4)(x+2)(x-3)=0$

$2x-4=0, x+2=0, x-3=0$

$x=2, x=-2, x=3$

d) $(x-2)(2x-1)(2x-4)=0$

$x-2=0, 2x-1=0, 2x-4=0$

$x=2, x=\frac{1}{2}, x=2$

Q5) a) Using synthetic division with $x=3$ gives

3	1	-8	19	-12
		3	-15	12
	1	-5	4	0

$x^3-8x^2+19x-12=0$

$(x-3)(x^2-5x+4)=0$

$(x-3)(x-4)(x-1)=0$

$x-3=0, x-4=0, x-1=0$

$x=3, x=4, x=1$

b) Using synthetic division with $x=1$ gives

1	3	4	-5	-2
		3	7	2
	3	7	2	0

$3x^3+4x^2-5x-2=0$

$(x-1)(3x^2+7x+2)=0$

$(x-1)(3x+1)(x+2)=0$

$x-1=0, 3x+1=0, x+2=0$