National 5 Mathematics

Trigonometry - Solutions

Marks are indicated in brackets after each question number

2014 Paper 1 Question 10, (2)

The graph has been stretched vertically by a factor of 3 so $a = 3$.

The graph has been moved to the right by 40° so $b = -40$.

2014 Paper 2 Question 12, (3)

$11\cos x° - 2 = 3$

$11\cos x° = 5$

$\cos x° = \frac{5}{11}$

$x = cos^{-1}\left(\frac{5}{11}\right) = 63°$

Second solution is $360 - 63 = 297°$.

2015 Paper 1 Question 6, (2)

Since the amplitude is 4, $a = 4$.

Since there are 3 copies of the Sine graph is $0 \leq x \leq 360°$, $b = 3$.

2015 Paper 1 Question 7, (1) (1) (1)

a) i) $a = -2$

ii) $b = -4$

b) $x = -4$

2015 Paper 1 Question 9, (2)

$\cos 90° = 0$

$\cos 100° < 0$ from inspection of the graph.

$\cos 300° > 0$ from inspection of the graph.

So, $\cos 100° < \cos 90° < \cos 300°$.

2016 Paper 1 Question 11, (2)

$tan^2x\ cos^2x$

Use $\tan x = \frac{\sin x}{\cos x}$ to give

$\left(\frac{\sin x}{\cos x}\right)^2 cos^2x$

$= \frac{sin^2x}{cos^2x} \cdot cos^2x$

$= sin^2x$

2016 Paper 2 Question 14, (3)

$2\tan x + 5 = -4$

$2\tan x = -9$

$\tan x = -4.5$

$x = tan^{-1}(-4.5)$

$x = 77°$

Using CAST we have

$x = 180 - 77 = 103°$

$x = 360 - 77 = 283°$

2017 Paper 2 Question 15, (1) (1) (4)

a) $h = 40 + 23\cos x$

When $x = 60°$

$h = 40 + 23\cos 60°$

$= 51.5\ m.$

b) Minimum height occurs where $x = 180°$

$h = 40 + 23\cos 180°$

$= 17\ m.$

c) Let $h = 61$ to give

$61 = 40 + 23\cos x$

$\cos x = \frac{21}{23}$

$x = cos^{-1}\left(\frac{21}{23}\right) = 24°$

$x = 360 - 24 = 336°$

2018 Paper 1 Question 6, (2)

$y = 5\cos 4x$

$a = 5, b = 4$

2018 Paper 1 Question 10, (3)

$z^2 = x^2 + y^2 - 2xy\cos Z$

$= 8^2 + 10^2 - 2(8)(10)(\frac{1}{8})$

$= 164 - 20$

$= 144$

$z = 12$

So, $XY = 12$ cm.

2018 Paper 1 Question 12, (1)

Sketch the graph of $y = \cos x$

Mark on a horizontal line through 0.5.

The line passes through the graph where $x = 60$.

From the symmetry of the graph, $\cos 240° = -0.5$.

Or use a CAST diagram.

2018 Paper 1 Question 18, (2)

$\sin x \cos x \tan x$

Substitute $\tan x = \frac{\sin x}{\cos x}$ to give

$\sin x \cos x \dfrac{\sin x}{\cos x}$

$= \sin x \sin x$

$= sin^2 x$

2018 Paper 2 Question 8, (3)

$7 \sin x + 2 = 3$

$\sin x = \frac{1}{7}$

$x = sin^{-1}\left(\frac{1}{7}\right) = 8.2°$

From CAST diagram

$x = 180 - 8.2 = 171.8°.$

2018 Paper 2 Question 17, (5)

Area of Triangle $= \frac{1}{2}(38)(55) \sin 75$

$= 1009.39\ cm^2.$

Area of Sector $= \frac{75}{360}\ x\ \pi\ x\ 60$

$= 39.27\ cm^2.$

Shaded Area $= 1009.39 - 39.27$

$= 970.12\ cm^2.$

2019 Paper 1 Question 13, (2)

$x -$ co-ordinate of $A = 180 - 45 = 135.$

$y -$ co-ordinate of $A = -1\ x\ 3 = -3.$

Co-ordinates of $A = (135, -3).$

2019 Paper 2 Question 7, (3)

The smallest angle is at vertex Z.

Using the Cosine Rule gives

$$Cos\ Z = \frac{8.5^2 + 7.2^2 - 6.3^2}{2\ (8.5)\ (7.2)}$$

$= \frac{84.4}{122.4}$

$Z = cos^{-1}\left(\frac{84.4}{122.4}\right)$

$= 46.4°.$

2019 Paper 2 Question 14, (3)

$5\cos x + 2 = 1$

$5\cos x = -1$

$\cos x = -\frac{1}{5}$

$\cos x = -0.2$

$cos^{-1}(0.2) = 78°$

From CAST solutions lie in quadrants 2 & 3, giving

$x = 180 - 78 = 102°$.

$x = 180 + 78 = 258°$.

2019 Paper 2 Question 17, (2)

$(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$

$= sin^2x + 2\sin x \cos x + cos^2x$

$= sin^2x + cos^2x + 2\sin x \cos x$

$[sin^2x + cos^2x = 1$ from Trig Identities$]$

$= 2\sin x \cos x + 1$

2022 Paper 1 Question 8, (1) (1)

a) $a = 3$

b) $b = 8$

2022 Paper 2 Question 9, (3)

$3\sin x + 4 = 6$

$3\sin x = 2$

$\sin x = \frac{2}{3}$

$sin^{-1}\left(\frac{2}{3}\right) = 41.8°$

From CAST $x = 41.8°$ and $x = 180° - 41.8° = 138.2°$

2022 Paper 2 Question 13, (2)

$$\frac{\sin x + 2\cos x}{\cos x}$$

$$= \frac{\sin x}{\cos x} + \frac{2\cos x}{\cos x}$$

$$= \tan x + 2$$

2023 Paper 1 Question 11, (1)

Either use the CAST diagram or the symmetry of the Sine Graph to answer this question.

$\sin 330° = -\sin 30° = -0.5$

2023 Paper 1 Question 13, (1) (1)

a) $a = 210$

b) $b = 1$

2023 Paper 2 Question 11, (4)

$h = 20\cos x + 147$

Substitute $h = 150$ to give

$150 = 20\cos x + 147$

$20\cos x = 3$

$$\cos x = \frac{3}{20}$$

$$cos^{-1}\left(\frac{3}{20}\right) = 81.4°$$

From CAST $x = 81.4°$ and $x = 360 - 81.4 = 278.6°$

2023 Paper 2 Question 13, (2)

$$2sin^2x + 2cos^2x = 2(sin^2x + cos^2x)$$

$$= 2(1)$$

$$= 2$$