National 5 Mathematics

Quadratic Equations - Solutions

Marks are indicated in brackets after each question number

2016 Paper 1 Question 6, (2)

$f(x) = 7x^2 + 5x - 1$

$a = 7, b = 5, c = -1$

$b^2 - 4ac = 25 - 4\ x\ 7\ x\ (-1) = 53$

Since $b^2 - 4ac > 0$ there are two roots.

2016 Paper 1 Question 12, (1) (3) (3)

a) Area of rectangle $= (2x + 1)(x + 8)$

b) Area of triangle $= \frac{1}{2}(3x)(2(x + 5))$

$= 3x(x + 5)$

Area of rectangle = area of triangle

$(2x + 1)(x + 8) = 3x(x + 5)$

$2x^2 + 17x + 8 = 3x^2 + 15x$

Simplifying gives $x^2 - 2x - 8 = 0$

c) $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = -2, x = 4$

Since x is a length it cannot be negative, so $x = 4$.

Length $= 8 + 4 = 12\ cm$.

Breadth $= (2\ x\ 4) + 1 = 9\ cm$.

2017 Paper 2 Question 4, (3)

$2x^2 + 5x - 4 = 0$

$a = 2, b = 5, c = -4$

$$x = \frac{-5 \pm \sqrt{5^2 - 4\ x\ 2x\ (-4)}}{2\ x\ 2}$$

$$x = \frac{-5 \pm \sqrt{25 + 32}}{4}$$

$$x = \frac{-5 + \sqrt{57}}{4} = 0.6 \qquad x = \frac{-5 - \sqrt{57}}{4} = -3.1$$

2018 Paper 1 Question 5, (2)

$x^2 - 11x + 24 = 0$

$(x - 8)(x - 3) = 0$

$x - 8 = 0$ and $x - 3 = 0$

$x = 8, x = 3$

2018 Paper 1 Question 8, (2)

$f(x) = 2x^2 + 4x + 5$

$a = 2, b = 4, c = 5$

$b^2 - 4ac = 4^2 - 4(2)(5)$

$= 16 - 40$

$= -24$

So, no real roots.

2018 Paper 1 Question 19, (4)

b) $x^2 - 6x - 81 = 0$

$(x - 3)^2 - 90 = 0$

$(x - 3)^2 = 90$

$x - 3 = \pm\sqrt{90}$

$x = 3 \pm 3\sqrt{10}$

2019 Paper 1 Question 15, (1) (4)

a) $h = 12t - 5t^2$

Substitute $t = 2$ to give

$h = (12\ x\ 2) - 5(2^2)$

$= 24 - 20$

$= 4$

4 metres.

b) Substitute $h = -17$ to give

$-17 = 12t - 5t^2$

$5t^2 - 12t - 17 = 0$

$(5t - 17)(t + 1) = 0$

$5t - 17 = 0$

$t = \frac{17}{5}$

$t = 3.4$

$t + 1 = 0$

$t = -1$

Since t represents *time* this solution can be discarded.

So, the ball will hit the sea after 3.4 seconds.

2019 Paper 2 Question 6, (3)

$3x^2 + 9x - 2 = 0$

$a = 3, b = 9, c = -2$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(3)(-2)}}{2(3)} = \frac{-9 \pm \sqrt{81 + 24}}{6}$$

$$= \frac{-9 \pm \sqrt{105}}{6}$$

$x = \frac{-9 + \sqrt{105}}{6} = 0.2$ $\qquad$ $x = \frac{-9 - \sqrt{105}}{6} = -3.2$

2022 Paper 1 Question 15, (1) (4)

a) Area of triangle $= \frac{1}{2}\ x\ base\ x\ height$

$$= \frac{1}{2}(3)(x+12)$$

$$= \frac{3}{2}(x+12)$$

b) Area of rectangle $= base\ x\ height$

$$= (8-x)\ x\ 6$$

$$= 6(8-x)$$

Area of triangle = Area of rectangle

$\frac{3}{2}(x+12) = 6(8-x)$

$\frac{3}{2}(x+12) = 48-6x$

$3(x+12) = 96-12x$

$3x+36 = 96-12x$

$15x = 60$

$x = 4$

2022 Paper 2 Question 7, (4)

$4x^2 + 2x - 7 = 0$

$a = 4, b = 2, c = -7$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{116}}{8}$$

$x = \frac{-2+\sqrt{116}}{8}$ and $x = \frac{-2-\sqrt{116}}{8}$

$x = 1.1$ and $x = -1.6$ to 2 significant figures.

2023 Paper 1 Question 5, (2)

$f(x) = 4x^2 + 6x - 1$

$a = 4, b = 6, c = -1$

$b^2 - 4ac = 6^2 - 4(4)(-1)$

$= 36 + 16$

$= 42$

Since $b^2 - 4ac > 0$ there are two real and distinct roots.

2023 Paper 2 Question 14, (2) (4)

a) Volume $= l\ x\ b\ x\ h$

$= (x + 7)(x)(2)$

$= 2x(x + 7)$

$= 2x^2 + 14x$

But since the volume is 45 we have

$2x^2 + 14x = 45$

$2x^2 + 14x - 45 = 0$

b) We need to solve $2x^2 + 14x - 45 = 0$

Since this doesn't factorise we have to use the quadratic formula

$a = 2, b = 14, c = -45$

$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(-45)}}{2(2)}$

$x = \frac{-14 \pm \sqrt{556}}{4}$

$x = \frac{-14 + \sqrt{556}}{4}$ and $x = \frac{-14 - \sqrt{556}}{4}$

$x = 2.4$ and $x = -9.4$

But since x is a length it cannot be negative, so $x = 2.4$.

So, the breadth is 2.4 cm.