National 5 Mathematics

Pythagoras' Theorem – Solutions

Marks are indicated in brackets after each question number

2014 Paper 2 Question 6, (4)

Since Lowtown is due west of Midtown then Hightown can only be directly north of Lowtown if the triangle is right-angled.

Let H = Hightown, L = Lowtown, M= Midtown

Then $(LH)^2 + (LM)^2 = 85^2 + 75^2 = 12,850$

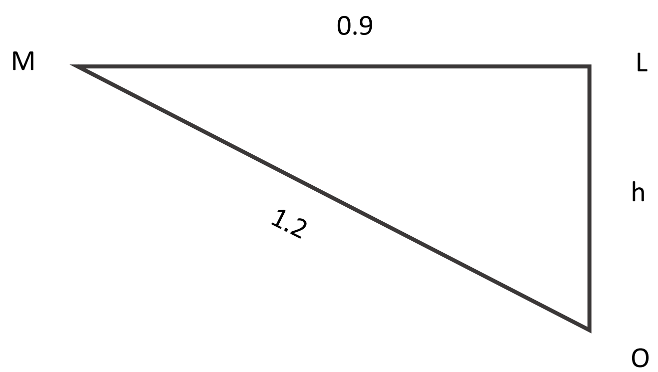
$(HM)^2 = 110^2 = 12,100$

Since $(LH)^2 + (LM)^2 \neq (HM)^2$ the triangle is not right-angled.

Therefore, Hightown is not directly north of Lowtown.

2015 Paper 2 Question 12, (4)

Construct a right triangle from the midpoint of ML with O & M.



Using Pythagoras gives

$1.2^2 = 0.9^2 + h^2$

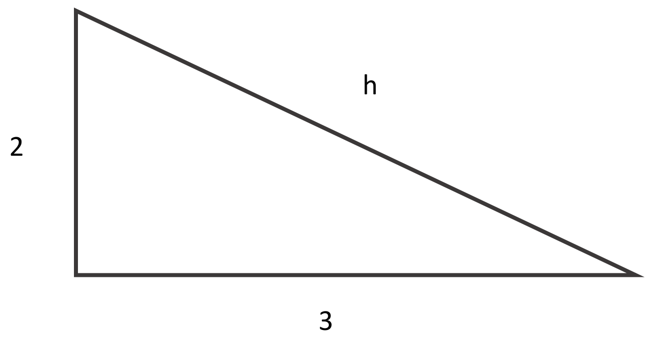
Solving gives $h = 0.79\ m$

So, depth of milk $= 0.79 + radius = 0.79 + 1.2 = 2.78\ m$.

2016 Paper 1 Question 7, (4)

a) $B = (8, 4, 0)$ by inspection of the graph.

b) Create a right-angled triangle in the base.



Using Pythagoras, we have

$h = \sqrt{2^2 + 3^2} = \sqrt{13}$

$(AV)^2 = 6^2 + (\sqrt{13})^2$

$= 49$

$AV = 7$

2016 Paper 2 Question 16, (4)

Using Pythagoras gives $DE = \sqrt{4^2 - 3^2} = \sqrt{7}$

Using the Sine Rule on ADE gives

$$\frac{\sin A}{a} = \frac{\sin E}{e}$$

$$\frac{\sin A}{\sqrt{7}} = \frac{\sin 90}{4}$$

$$\sin A = \frac{\sqrt{7}\sin 90}{4}$$

$= 0.661 \ldots$

$A = sin^{-1}(0.661 \ldots)$

$= 41°$

Using the Cosine Rule on ABC gives

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 6^2 + 10^2 - 2\ x\ 6\ x\ 10\ x\cos 41°$

$= 45.4$

$a = 6.7$

So, $BC = 6.7\ cm$.

2017 Paper 2 Question 7, (3)

The hypotenuse of the larger triangle is 22 cm.

The short sides have length 8 cm and 19 cm.

$8^2 + 19^2 = 425$

$22^2 = 484$

Since $425 \neq 484$ the triangle is not right angled by the converse of Pythagoras.

2018 Paper 2 Question 12, (4)

Let M be the mid-point of AB.

Construct a right-angled triangle OAM.

Using Pythagoras, $13^2 - 10^2 = 169 - 100 = 69$.

$\sqrt{69} = 8.3$

Width = Radius + 8.3 = 13 + 8.3 = 21.3 cm.

2019 Paper 2 Question 11, (4)

The length of B to C is given by $1500 - 600 - 650 = 250\ m$.

$650^2 = 422{,}500$

$600^2 + 250^2 = 422{,}500$

Since $600^2 + 250^2 = 650^2$ a triangle with short sides 600 & 250 and long side 650 is a right-angled triangle by the Converse of Pythagoras' Theorem.

So, ABC is a right-angled triangle, meaning that B is due east of A since C is due north of B.

2019 Paper 2 Question 18, (4)

Create a right angled triangle TSB.

Since TS & SB are the radius of the circle they have length 7.5 cm.

By Pythagoras, $TB = \sqrt{7.5^2 + 7.5^2}$

$= 10.6\ cm$

TB is the radius of the larger circle, so TD also has length 10.6 cm.

So, height = $10.6 + 15 = 25.6\ cm$.

2022 Paper 2 Question 8, (4)

Set up a right angled triangle with long side $= 2.9\ m$ and short side $= 2\ m$.

By Pythagoras we have

$a^2 + 2^2 = 2.9^2$

$a^2 + 4 = 8.41$

$a^2 = 4.41$

$a = \sqrt{4.41} = 2.1$

Height $= 2.1 + radius$

$= 2.1 + 2.9 = 5m$

2022 Paper 2 Question 11, (3)

Set up a right angled triangle on the base of the cuboid, EGH.

This triangle has short sides 24 cm and 6 cm. Let the long side be c.

Using Pythagoras gives

$24^2 + 6^2 = c^2$

$576 + 36 = c^2$

$c^2 = 612$

$c = 24.7\ cm$

Set up a second right angled triangle which includes the diagonal, ECG.

This triangle has short sides 24.7 cm and 8 cm. Let the long side be c.

Using Pythagoras gives

$24.7^2 + 8^2 = c^2$

$610.1 + 64 = c^2$

$c^2 = 674.1$

$c = 26\ cm$

2023 Paper 1 Question 10, (4)

Consider a right angled triangle from the midpoint of AB to A to C.

This triangle has a short side of 30 cm, and a long side of 50 cm. Let the other short side be a. Then, using Pythagoras we have,

$a^2 + 30^2 = 50^2$

$a^2 + 900 = 2500$

$a^2 = 1600$

$a = 40\ cm$

So, the width $= 40 + radius$

$= 40 + 50$

$= 90\ cm$

2023 Paper 2 Question 8, (4)

The wall is perpendicular (i.e. right-angled) to the ground if the triangle ABC is right-angled.

$4^2 + 7^2 = 16 + 49 = 65$

$8^2 = 64$

Since $65 \neq 64$, by the converse of Pythagoras Theorem, the triangle is not right-angled.

Therefore, the wall is not perpendicular to the ground.