Series Convergence Tests

The Integral Convergence Test

Let f be a function defined on the interval $[k, \infty)$, where k is an integer. Then the infinite series $\sum_{n=k}^{\infty} f(n)$ converges to a real number if and only if the improper integral $\int_k^{\infty} f(x)\, dx$ is finite. If the integral diverges then the series also diverges.

The P-Series Test

The series $\sum_{n=k}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

In the case where $p = 1$ we have the harmonic series which diverges.

The Nth Term Test

If $\lim_{n \to \infty} a_n \neq 0$, or if this limit does not exist, then the infinite series $\sum_{n=1}^{\infty} a_n$ diverges.

It is important to note that the Nth term test is a divergence, not convergence, test.

The Direct Comparison Test

If the series $\sum_{n=1}^{\infty} b_n$ is a absolutely convergent and $|a_n| \leq |b_n|$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely. Also, if $\sum_{n=1}^{\infty} a_n$ diverges then $\sum_{n=1}^{\infty} b_n$ diverges.

The Limit Comparison Test

Assume the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are such that $a_n > 0, b_n > 0$ for all n.

Then if $\lim_{n \to \infty} \frac{a_n}{b_n}$ exists, then either both series converge or both series diverge.

The Ratio Test

Let $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$. Then if $L < 1$, the series converges absolutely. If $L > 1$, the series diverges. If $L = 1$, or if the limit fails to exist, the ratio test is inconclusive and another test must be used.

The Root Test

Let $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L$. Then if $L < 1$, the series converges. If $L > 1$, the series diverges. If $L = 1$, or if the limit fails to exist, the ratio test is inconclusive and another test must be used.

The Alternating Series Test

If $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \le a_n$ then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.