Integration Formula List

Power Rule

$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$

Integration by Parts

If u & v are functions of x then

$$\int u \, dv = uv - \int v \, du$$

$$\int_a^b u \, dv = [uv]_a^b - \int_a^b v \, du$$

Reverse Chain rule

$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

Trigonometric Functions

$\int \sin(ax) \, dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax) \, dx = \frac{1}{a}\cos(ax) + c$

$\int \tan(ax) \, dx = -\frac{1}{a}\log\cos(ax) + c$

Inverse and Reciprocal Trigonometric Functions

$$\int sec(ax) \, dx = \frac{1}{a} log(sec(ax) + tan(ax)) + c$$

$$\int cosec(ax) \, dx = \frac{1}{a} log(cosec(ax) - cot(ax)) + c$$

$$\int cot(ax) \, dx = \frac{1}{a} log(sin(ax)) + c$$

$$\int sin^{-1}(ax)\, dx = x\left(sin^{-1}(ax)\right) + \frac{\sqrt{1-a^2x^2}}{a}$$

$$\int cos^{-1}(ax)\, dx = x\left(cos^{-1}(ax)\right) - \frac{\sqrt{1-a^2x^2}}{a}$$

Exponential Functions

$$\int e^x\, dx = e^x + c$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + c$$

Volumes of Revolution

Volume of revolution around the x-axis

$$V_x = \pi \int_a^b y^2\, dx$$

Volume of revolution around the y-axis

$$V_y = \pi \int_a^b x^2\, dy$$

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \le t \le b$ the volume of revolution around the x axis is given by

$$V_x = \pi \int_a^b y^2\, \frac{dx}{dt}\, dt$$

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \le t \le b$ the volume of revolution around the y axis is given by

$$V_y = \pi \int_a^b x^2\, \frac{dy}{dt}\, dt$$

Surface Area of Revolution

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \leq t \leq b$ the surface area of revolution around the x axis is given by

$$A_x = 2\pi \int_a^b y \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$$

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \leq t \leq b$ the surface area of revolution around the y axis is given by

$$A_y = 2\pi \int_a^b x \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$$

For the polar curve $r = f(\theta)$, where $\alpha < \theta < \beta$, the surface area of revolution formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the polar axis is

$$A = 2\pi \int_\alpha^\beta f(\theta) \sin\theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$$

For the polar curve $r = f(\theta)$, where $\alpha < \theta < \beta$, the surface area of revolution formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the line $\theta = \frac{\pi}{2}$

$$A = 2\pi \int_\alpha^\beta f(\theta) \cos\theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$$

Arc Length

For functions given as $y = f(x)$ with limits $[x_A, x_B]$

$$S = \int_{x_A}^{x_B} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

For functions given as $x = f(y)$ with limits $[y_A, y_B]$

$$S = \int_{y_A}^{y_B} \sqrt{1 + (\frac{dx}{dy})^2}\, dy$$

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \leq t \leq b$
the length of the arc between a and b is given by

$$S = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\ dt$$

For the polar curve $r = f(\theta)$, where $\alpha < \theta < \beta$, the length of the arc between α and β is given by

$$S = \int_\alpha^\beta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\ d\theta = \int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2}\ d\theta$$

Area Under a Curve

For the curve given by $y = f(x)$ the area under the curve and between the x-axis and the limits a and b is given by

$$A = \int_a^b f(x)\ dx$$

For a curve defined parametrically by $x(t)$ and $y(t)$ on the interval $a \leq t \leq b$
the area under the curve and between the x-axis and the limits a and b is given by

$$A = \int_a^b y(t)\ x'(t)\ dx$$

For the polar curve $r = f(\theta)$, where $\alpha < \theta < \beta$, the area is given by

$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\ d\theta$ or $A = \frac{1}{2}\int_\alpha^\beta r^2\ d\theta$