Differentiation Formula List

Power Rule

$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$ or $\frac{d}{dx}[u^n] = nu^{n-1}u'$

$\frac{d}{dx}[cu] = cu'$

Product & Quotient Rule

$$\frac{d}{dx}[uv] = uv' + vu'$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

Chain rule

The chain rule is used to differentiate composite functions which may be presented in many different forms.

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ or for $h = f\big(g(x)\big), h'(x) = f'\big(g(x)\big)g'(x)$

Trigonometric Functions

$\frac{d}{dx}[\sin u] = (\cos u)u'$

$\frac{d}{dx}[\cos u] = -(\sin u)u'$

Inverse and Reciprocal Trigonometric Functions

$$\frac{d}{dx}[sin^{-1} u] = \frac{u'}{\sqrt{1 - u^2}}$$

$$\frac{d}{dx}[\sec u] = (\sec u \tan u)\, u'$$

$$\frac{d}{dx}[cos^{-1} u] = \frac{-u'}{\sqrt{1 - u^2}}$$

$$\frac{d}{dx}[\operatorname{cosec} x] = -\,(cosec\ u \cot u)u'$$

$$\frac{d}{dx}[tan^{-1} u] = \frac{u'}{1 + u^2}$$

$$\frac{d}{dx}[\cot x] = -\,(cosec^2 u)u'$$

$$\frac{d}{dx}[sec^{-1} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$$

$$\frac{d}{dx}[cosec^{-1} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$$

$$\frac{d}{dx}[cot^{-1} u] = \frac{-u'}{1 + u^2}$$

Exponential Functions

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^u = e^u u'$$

Logarithmic Functions

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

Parametric Differentiation

If x and y are functions of a parameter t, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right)}{dx/dt}$$

Polar Differentiation

$r = f(\theta)$ can be written in parametric form as

$x = r\cos\theta = f(\theta)\cos\theta$ and $y = r\sin\theta = f(\theta)\sin\theta$

Then, by parametric differentiation, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

Rolle's Theorem

If a real valued function f is continuous on a closed interval $[a, b]$, differentiable on the open interval (a, b), and $f(a) = f(b)$, then there exists at least one value c in the open interval (a, b) such that $f\,'(c) = 0$.

The Mean Value Theorem

If a real valued function f is continuous on a closed interval $[a, b]$, differentiable on the open interval (a, b), then there exists a point c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

L'Hospital's Rule

For two functions $f(x)$ and $g(x)$, if either

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$

Then provided that $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$