Applications

Solutions

Finding the Area Under a Curve

Finding the Area Between Two Curves

Volumes of Revolution

Arc Length

Surface of Revolution

Finding the Area Under a Curve

Q1) a) $\int_1^4 3x^2 - 6x + 4\ dx = [x^3 - 3x^2 + 4x]_1^4$

$= (4^3 - 3 \cdot 4^2 + 4 \cdot 4) - (1^3 - 3 \cdot 1^2 + 4 \cdot 1)$

$= 30$

Area $= 30\ units^2$

b) $\int_2^4 3x^2 - 12x + 12\ dx = [x^3 - 6x^2 + 12x]_2^4$

$= (4^3 - 6 \cdot 4^2 + 12 \cdot 4) - (2^3 - 6 \cdot 2^2 + 12 \cdot 2)$

$= (64 - 96 + 48) - (8 - 24 + 24)$

$= 8$

Area $= 8\ units^2$

c) $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 3\sin(2x)\ dx = [-\frac{3}{2}\cos(2x)]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$

$= (-\frac{3}{2}\cos\left(\frac{10\pi}{6}\right)) - (-\frac{3}{2}\cos\left(\frac{2\pi}{2}\right))$

$= (-\frac{3}{2}\cos\left(\frac{5\pi}{3}\right)) - (-\frac{3}{2}\cos(\pi))$

$= (-0.75) - (1.5)$

$= -2.25$

Area $= 2.25\ units^2$

d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\cos x\, dx = [2\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = (2\sin\frac{\pi}{4}) - (2\sin\frac{\pi}{6})$

$= 1.414 - 1$

$= 0.414$

Area $= 0.414\ units^2$

Q2) a) $y = 10 + 3x - x^2$

For $x-$ intercepts let $y = 0$ to give

$10 + 3x - x^2 = 0$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = -2, x = 5$

b) $\int_{-2}^{5} 10 + 3x - x^2\, dx$

$= [10x + \frac{3}{2}x^2 - \frac{1}{3}x^3]_{-2}^{5}$

$= \left(10\cdot 5 + \frac{3}{2}\cdot 5^2 - \frac{1}{3}\cdot 5^3\right) - (10\cdot(-2) + \frac{3}{2}(-2)^2 - \frac{1}{3}(-2)^3)$

$= (45.8) - (-11.3)$

$= 57.1$

Area $= 57.1\ units^2$

Q3) $x = \mathrm{t}(1+\mathrm{t}) = t + t^2, \frac{dx}{dt} = 1 + 2t$

When $x = 0, \mathrm{t}(1+\mathrm{t}) = 0$ so $t = 0$ or $t = -1$

When $x = 0, \mathrm{t}(1+\mathrm{t}) = 2$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2,\ t = 0$

Since $t \geq 0$ we use $t = 0$ and $t = 1$

Area $= \int_0^1 y\, \frac{dx}{dt}\, dt = \int_0^1 \frac{1}{1+t}\,(1+2t)\, dt$

$= \int_0^1 (2 - \frac{1}{1+t})\, dt$

$= [2x - \ln|1+t|]_0^1$

$= (2 - \ln 2) - (-\ln 1)$

$= 2 - \ln 2$

Q4) $r = a\cos\theta\,, \alpha = 0\,, \beta = \frac{\pi}{2}$

Area $= \frac{1}{2}a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

$= \frac{a^2}{4}\int_0^{\frac{\pi}{2}} (\cos 2\theta + 1)\, d\theta$

$= \frac{a^2}{4}\left[\frac{1}{2}\sin 2\theta + \theta\right]_0^{\frac{\pi}{2}}$

$= \frac{a^2}{4}\left[\left(0 + \frac{\pi}{2}\right) - (0)\right]$

$= \frac{\pi a^2}{8}$

Finding the Area Between Two Curves

Q1) For points of intersection

$x^2 - 4x + 5 = 2x - 3$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 2,\ x = 4$

$\int_2^4 [(2x - 3) - (x^2 - 4x + 5)]\, dx$

$= \int_2^4 (-x^2 + 6x - 8)\, dx$

$= [-\frac{1}{3}x^3 + 3x^2 - 8x]_2^4$

$= \left(-\frac{1}{3}\cdot 4^3 + 3\cdot 4^2 - 8\cdot 4\right) - (-\frac{1}{3}\cdot 2^3 + 3\cdot 2^2 - 8\cdot 2)$

$= \left(-\frac{64}{3} + 48 - 32\right) - (-\frac{8}{3} + 12 - 16)$

$= \frac{4}{3}$

Area $= \frac{4}{3}\ units^2$

Q2) For points of intersection

$2x^2 - 4x - 15 = -x^2 + 2x + 9$

$3x^2 - 6x - 24 = 0$

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = -2, x = 4$$

$$\int_{-2}^{4}[(-x^2 + 2x + 9) - (2x^2 - 4x - 15)]\,dx$$
$$= \int_{-2}^{4}(-3x^2 + 6x + 24)\,dx$$
$$= [-x^3 + 3x^2 + 24x]_{-2}^{4}$$
$$= (-4^3 + 3\cdot 4^2 + 24\cdot 4) - (-(-2)^3 + 3(-2)^2 + 24(-2))$$
$$= (-64 + 48 + 96) - (8 + 12 - 48)$$
$$= 108$$
$$\text{Area} = 108\ units^2$$

Q3) For points of intersection

$$-x^2 + 4x - 2 = x^2 + 3x - 8$$
$$-2x^2 + x + 6 = 0$$
$$2x^2 - x - 6 = 0$$
$$(2x + 3)(x - 2) = 0$$
$$x = -1.5, x = 2$$

$$\int_{-1.5}^{2}[(-x^2 + 4x - 2) - (x^2 + 3x - 8)]\,dx$$
$$= \int_{-1.5}^{2}(-2x^2 + x + 6)\,dx$$
$$= [-\tfrac{2}{3}x^3 + \tfrac{1}{2}x^2 + 6x]_{-1.5}^{2}$$
$$= \left(-\tfrac{2}{3}\cdot 2^3 + \tfrac{1}{2}\cdot 2^2 + 6\cdot 2\right) - (-\tfrac{2}{3}(-1.5)^3 + \tfrac{1}{2}(-1.5)^2 + 6(-1.5))$$
$$= \left(-\tfrac{16}{3} + 2 + 12\right) - (2.25 + 1.125 - 9)$$
$$= 14.29$$
$$\text{Area} = 14.29\ units^2$$

Volumes of Revolution

Q1) a) $y = 10x^2$ between $x = 0$ and $x = 2$

$\pi \int_0^2 (10x^2)^2 \, dx$

$= \pi \int_0^2 100x^4 \, dx$

$= \pi \, [20x^5]_0^2$

$= \pi((20 \, x \, 2^5) - (20 \, x \, 0^5))$

$= 640\pi$

Volume $= 640\pi$

b) $y = \sqrt{x}$ between $x = 2$ and $x = 10$

$\pi \int_2^{10} \left(\sqrt{x}\right)^2 \, dx$

$= \pi \int_2^{10} x \, dx$

$= \pi \left[\frac{1}{2}x^2\right]_2^{10}$

$= \pi((50) - (2))$

$= 48\pi$

Volume $= 48\pi$

Q2) a) $y = \frac{2}{x+1}$ between $x = 0$ and $x = 2$

$\pi \int_0^2 (\frac{2}{(x+1)})^2 \, dx$

$= \pi \int_0^2 \frac{4}{(x+1)^2} dx = \pi \int_0^2 4(x+1)^{-2} \, dx$

$= \pi \left[-\frac{4}{x+1}\right]_0^2$

$= \pi \left(\left(-\frac{4}{3}\right) - (-4)\right)$

$= \frac{8\pi}{3}$

b) $y = \sin 2x$ between $x = 0$ and $x = \frac{\pi}{2}$

$\pi \int_0^{\frac{\pi}{2}} (\sin 2x)^2 \, dx = \pi \int_0^{\frac{\pi}{2}} sin^2 2x \, dx$

Using the trig identity $\cos 4x = 1 - 2sin^2 2x$ we have

$\pi \int_0^{\frac{\pi}{2}} (\frac{1}{2} - \frac{1}{2}\cos 4x) \, dx$

$= \pi \left[\frac{1}{2}x - \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{2}}$

$= \pi\left(\left(\frac{\pi}{4} - \frac{1}{8}\sin 2\pi\right) - \left(0 - \frac{1}{8}\sin 0\right)\right)$

$= \pi\left(\left(\frac{\pi}{4}\right) - (0)\right)$

$= \frac{\pi^2}{4}$

Volume $= \frac{\pi^2}{4}$

Q3) a) $y = \frac{1}{x}$ between $y = 1$ and $y = 3$

Rearranging gives

$x = \frac{1}{y} = y^{-1}$

$\pi \int_1^3 y^{-2}\, dy$

$= \pi \left[-\frac{1}{y}\right]_1^3$

$= \pi\left(\left(-\frac{1}{3}\right) - (-1)\right)$

$= \frac{2}{3}\pi$

Volume $= \frac{2}{3}\pi$

b) $y = 2e^{x^2}$ between $y = 2$ and $y = 4$

Rearranging gives

$e^{x^2} = \frac{y}{2}$

$\ln e^{x^2} = \ln \frac{y}{2}$

$x^2 = \ln\left(\frac{y}{2}\right)$

$\pi \int_2^4 \ln(\frac{y}{2})\, dy$

$= \pi \int_2^4 (\ln y - \ln 2)\, dy$

$= \pi(\int_2^4 \ln y\, dy - \int_2^4 \ln 2\, dy)$

$\ln y$ has to be integrated using integration by parts

Let $u = \ln y$ and let $\frac{dv}{dy} = 1$

$\frac{du}{dy} = \frac{1}{y}$ and $v = y$

$\int_2^4 \ln y \, dy = [y \ln y]_2^4 - \int_2^4 1 \, dy$

$= (4 \ln 4 - 2 \ln 2) - [y]_2^4$

$= 6 \ln 2 - 2$ Using logarithm rules

So, $\pi\left(\int_2^4 \ln y \, dy - \int_2^4 \ln 2 \, dy\right) = \pi(6 \ln 2 - 2 - \int_2^4 \ln 2 \, dy)$

$= \pi(6 \ln 2 - 2 - [y \ln 2]_2^4)$

$= \pi(6 \ln 2 - 2 - (4 \ln 2 - 2 \ln 2))$

$= \pi(4 \ln 2 - 2)$

Volume $= \pi(4 \ln 2 - 2)$

Q4) $x = e^t, \frac{dx}{dt} = e^t$

$y = \sqrt{t-1}, y^2 = t - 1$

When $x = e^2, t = 2$

When $x = e^3, t = 3$

Volume $= \pi \int_2^3 (t-1)(e^t) \, dt$

$= \pi \int_2^3 (te^t - e^t) \, dt = \pi(\int_2^3 te^t \, dt - [e^t]_2^3)$

$= \pi(\int_2^3 te^t \, dt - (e^3 - e^2))$

$= \pi(\int_2^3 te^t \, dt - e^3 + e^2)$

To evaluate $\int te^t \, dt$ we have to use integration by parts.

Let $u = t$ and let $\frac{dv}{dt} = e^t$

So, $\frac{du}{dt} = 1, du = dt$ and $v = e^t$

$\int_2^3 te^t \, dt = [te^t]_2^3 - \int_2^3 e^t \, dt = (3e^3 - 2e^2) - [e^t]_2^3$

$= (3e^3 - 2e^2) - (e^3 - e^2)$

$= 2e^3 - e^2$

Volume $= \pi\left((2e^3 - e^2) - e^3 + e^2\right) = \pi e^3$

Arc Length

Q1) a) $y = \frac{2}{3}x^{\frac{3}{2}}$ between $x = 8$ and $x = 15$

$\frac{dy}{dx} = x^{\frac{1}{2}}, (\frac{dy}{dx})^2 = x$

$\int_8^{15} \sqrt{1+x}\, dx$

$= \int_8^{15} (1+x)^{\frac{1}{2}}\, dx$

$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_8^{15}$

$= \left(\frac{2}{3}\ x\ 64\right) - (\frac{2}{3}\ x\ 27)$

$= \frac{74}{3}$

Arc Length $= \frac{74}{3}$

b) $y^2 = \frac{4}{9}x^3$ between $y = 0$ and $y = 2\sqrt{3}$

Rearranging gives

$y = \sqrt{\frac{4}{9}x^3} = \frac{2}{3}x^{\frac{3}{2}}$

Changing the limits to x gives $x = 0$ and $x = 3$

$\frac{dy}{dx} = x^{\frac{1}{2}}$

$(\frac{dy}{dx})^2 = x$

$\int_0^3 \sqrt{1+x}\, dx$

$= \int_0^3 (1+x)^{\frac{1}{2}}\, dx$

$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^3$

$= \left(\frac{16}{3}\right) - (\frac{2}{3}) = \frac{14}{3}$

Arc Length $= \frac{14}{3}$

c) $x = \ln\cos y$ between $y = 0$ and $y = \frac{\pi}{3}$

$\frac{dx}{dy} = \frac{-\sin y}{\cos y} = -\tan y$

$(\frac{dx}{dy})^2 = tan^2 y$

$\int_0^{\frac{\pi}{3}} \sqrt{1+tan^2 y}\, dy$

$= \int_0^{\frac{\pi}{3}} \sqrt{sec^2 y}\, dy$ Since $sec^2 y = 1 + tan^2 y$

$= \int_0^{\frac{\pi}{3}} \sec y\ dy$

$= [\ln|\sec y + \tan y|]_0^{\frac{\pi}{3}}$

$= (\ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right|) - (\ln|\sec 0 + \tan 0|)$

$= (\ln|2+\sqrt{3}|) - (\ln 1)$

$= (\ln|2+\sqrt{3}|)$

Arc Length $= (\ln|2+\sqrt{3}|)$

Q2) a) $x = t^2, \frac{dx}{dt} = 2t, (\frac{dx}{dt})^2 = 4t^2$

$y = 2t, \frac{dy}{dt} = 2, (\frac{dy}{dt})^2 = 4$

$\int_0^2 \sqrt{4t^2+4}\, dt = \int_0^2 \sqrt{4(t^2+1)}\, dt$

$= 2\int_0^2 \sqrt{t^2+1}\ dt$

Let $t = \sinh u, \frac{dt}{du} = \cosh u, dt = \cosh u\, du$

When $t = 0, u = arsinh\ 0 = 0$

When $t = 2, u = arsinh\ 2$

$2\int_0^2 \sqrt{t^2+1}\ dt = 2\int_0^{arsinh\ 2} \sqrt{sinh^2 u + 1}\cosh u\, du$

$= 2\int_0^{arsinh\ 2} cosh^2 u\, du$

Use $\cosh 2u = 2cosh^2 u - 1$

$= \int_0^{arsinh\ 2} (1 + \cosh 2u)\ du$

$= \left[u + \frac{1}{2}\sinh 2u\right]_0^{arsinh\ 2}$

Use $\sinh 2u = 2\sinh u \cosh u$

$= [u + \sinh u \cosh u]_0^{arsinh\ 2}$

$= (arsinh\ 2 + 2\cosh(arsinh 2)) - (0 + \frac{1}{2}\sinh 0)$

$= arsinh\ 2 + 2\cosh(arsinh 2)$

$= \ln(2+\sqrt{5}) + 2\cosh(\ln(2+\sqrt{5}))$

$= \ln(2+\sqrt{5}) + 2\sqrt{5}$

b) $x = 3 + 2\cos t, \frac{dx}{dt} = -2\sin t, \left(\frac{dx}{dt}\right)^2 = 4sin^2t$

$y = 2t + 2\sin t, \frac{dy}{dt} = 2 + 2\cos t, (\frac{dy}{dt})^2 = 4cos^2t + 8\cos t + 4$

$\left(\frac{dx}{dt}\right)^2 + (\frac{dy}{dt})^2 = 4sin^2t + 4cos^2t + 8\cos t + 4$

$= 4 + 8\cos t + 4$

$= 8\cos t + 8$

Use $\cos 2t = 2\,cos^2t - 1$ written as $\cos t = 2cos^2\left(\frac{t}{2}\right) - 1$

$= 8\left(2cos^2\left(\frac{t}{2}\right) - 1\right) + 8$

$= 16\,cos^2\left(\frac{t}{2}\right)$

$\int_{\frac{\pi}{3}}^{\pi} \sqrt{16\,cos^2\left(\frac{t}{2}\right)}\,dt = 4\int_{\frac{\pi}{3}}^{\pi} \cos\frac{t}{2}\,dt$

$= 4\left[\sin\frac{t}{2}\right]_{\frac{\pi}{3}}^{\pi}$

$= 8(\sin\frac{\pi}{2} - \sin\frac{\pi}{6}) = 8\left(1 - \frac{1}{2}\right) = 4$

Q3) $r = a, \frac{dr}{d\theta} = 0$

Arc Length $= \int_0^{2\pi} \sqrt{a^2 + 0^2}\,d\theta$

$= \int_0^{2\pi} a\,d\theta$

$= [ax]_0^{2\pi}$

$= 2\pi a - 0$

$= 2\pi r$

Q4) $r = 5e^{2\theta}, \frac{dr}{d\theta} = 10e^{2\theta}$

Arc Length $= \int_0^{\frac{\pi}{2}} \sqrt{(5e^{2\theta})^2 + (10e^{2\theta})^2}\,d\theta$

$= \int_0^{\frac{\pi}{2}} \sqrt{25e^{4\theta} + 100e^{4\theta}}\,d\theta$

$= \sqrt{125}\int_0^{\frac{\pi}{2}} e^{2\theta}\,d\theta$

$= \sqrt{125}\left[\frac{1}{2}e^{2\theta}\right]_0^{\frac{\pi}{2}}$

$= \frac{5\sqrt{5}}{2}(e^{\pi} - 1)$

Surface of Revolution

Q1) $x = t^2, \frac{dx}{dt} = 2t$

$y = t - \frac{1}{3}t^3, \frac{dy}{dt} = 1 - t^2$

$$\text{Area} = 2\pi \int_0^1 y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^1 \left(t - \frac{1}{3}t^3\right)\sqrt{4t^2 + (1 - t^2)^2}\, dt$$

$$= 2\pi \int_0^1 \left(t - \frac{1}{3}t^3\right)\sqrt{4t^2 + 1 - 2t^2 + t^4}\, dt$$

$$= 2\pi \int_0^1 \left(t - \frac{1}{3}t^3\right)\sqrt{t^4 + 2t^2 + 1}\, dt$$

$$= 2\pi \int_0^1 \left(t - \frac{1}{3}t^3\right)\sqrt{(t^2 + 1)^2}\, dt$$

$$= 2\pi \int_0^1 \left(t - \frac{1}{3}t^3\right)(t^2 + 1)\, dt$$

$$= 2\pi \int_0^1 t + \frac{2}{3}t^3 - \frac{1}{3}t^5 dt$$

$$= 2\pi \left[\frac{1}{2}t^2 + \frac{1}{6}t^4 - \frac{1}{18}t^6\right]_0^1$$

$$= 2\pi(\frac{1}{2} + \frac{1}{6} - \frac{1}{18})$$

$$= 2\pi(\frac{11}{18})$$

$$= \frac{11\pi}{9}$$

Q2) $x = \sin^2 t = (\sin t)^2, \frac{dx}{dt} = 2\sin t \cos t$

$y = \cos^2 t = (\cos t)^2, \frac{dy}{dt} = -2\cos t \sin t$

$$\text{Area} = 2\pi \int_0^{\frac{\pi}{2}} x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \sin^2 t\sqrt{(2\sin t\cos t)^2 + (-2\cos t \sin t)^2}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{2}} sin^2\, t\sqrt{2}(2\, sin\, t\, cos\, t)\, dt$$ after simplification

$$= 4\sqrt{2}\,\pi \int_0^{\frac{\pi}{2}} cos\, t\, sin^3\, t\, dt$$

$$= 4\sqrt{2}\,\pi \left[\frac{sin^4\, t}{4}\right]_0^{\frac{\pi}{2}}$$

$$= 4\sqrt{2}\,\pi\, (\frac{1}{4}sin^4\left(\frac{\pi}{2}\right) - \frac{1}{4}sin^4(0))$$

$= 4\sqrt{2}\,\pi\,(\frac{1}{4} - 0)$

$= \sqrt{2}\,\pi$

Q3) $S = 2\pi \int_0^{\pi} r\cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$\frac{dr}{d\theta} = -\sin\theta$

$S = 2\pi \int_0^{\pi} \cos^2\theta \sqrt{\cos^2\theta + \sin^2\theta}\, d\theta$

$= 2\pi \int_0^{\pi} \cos^2\theta\, d\theta$

$= 2\pi \int_0^{\pi} (\cos 2\theta + 1) d\theta$

$= 2\pi \left[\frac{1}{2}\sin 2\theta + \theta\right]_0^{\pi}$

$= 2\pi((0 + \frac{\pi}{2}) - (0 + 0))$

$= \pi^2$

Q4) $S = 2\pi \int_0^{\frac{\pi}{2}} r\cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$\frac{dr}{d\theta} = e^{\theta}$

$S = 2\pi \int_0^{\frac{\pi}{2}} e^{\theta}\cos\theta \sqrt{e^{2\theta} + e^{2\theta}} d\theta$

$= 2\pi \int_0^{\frac{\pi}{2}} e^{\theta}\cos\theta \sqrt{2e^{2\theta}} d\theta$

$= 2\pi \int_0^{\frac{\pi}{2}} e^{\theta}\cos\theta \sqrt{2}\sqrt{e^{2\theta}} d\theta$

$= 2^{\frac{3}{2}}\pi \int_0^{\frac{\pi}{2}} e^{\theta}\cos\theta\,(e^{\theta}) d\theta$

$= 2^{\frac{3}{2}}\pi \int_0^{\frac{\pi}{2}} e^{2\theta}\cos\theta\, d\theta$

Let $I = \int_0^{\frac{\pi}{2}} e^{2\theta}\cos\theta\, d\theta$ and use integration by parts

Let $u = e^{2\theta}, \frac{du}{d\theta} = 2e^{2\theta}, du = 2e^{2\theta}\, d\theta$

Let $\frac{dv}{d\theta} = -\cos\theta\,, v = \sin\theta$

$I = \int_0^{\frac{\pi}{2}} e^{2\theta}\cos\theta\, d\theta$

$= e^{2\theta}\sin\theta - \int 2e^{2\theta}\sin\theta\, d\theta$

Repeat integration by parts to evaluate $\int 2e^{2\theta} \sin\theta \, d\theta$ giving

$= e^{2\theta} \sin\theta + 2e^{2\theta} \cos\theta - \int 4e^{2\theta} \cos\theta \, d\theta$

$= e^{2\theta} \sin\theta + 2e^{2\theta} \cos\theta - 4\int e^{2\theta} \cos\theta \, d\theta$

$= e^{2\theta} \sin\theta + 2e^{2\theta} \cos\theta - 4I$

So, $I = \frac{e^{2\theta}}{5}(\sin\theta + 2\cos\theta)$

$S = 2\pi \int_0^{\frac{\pi}{2}} e^{\theta} \cos\theta \sqrt{e^{2\theta} + e^{2\theta}} d\theta$

$= 2^{\frac{3}{2}}\pi \int_0^{\frac{\pi}{2}} e^{2\theta} \cos\theta \, d\theta$

$= 2^{\frac{3}{2}}\pi \left[\frac{e^{2\theta}}{5}(\sin\theta + 2\cos\theta)\right]_0^{\frac{\pi}{2}}$

$= \frac{2^{\frac{3}{2}}\pi}{5}(e^{\pi} - 2)$