Applications

Questions

Finding the Area Under a Curve

Finding the Area Between Two Curves

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Arc Length

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Finding the Area Under a Curve

Q1) Find the area between the given curve, the $x-$ axis, and the limits.

a) $y = 3x^2 - 6x + 4$ $\quad x = 1, x = 4$

b) $y = 3x^2 - 12x + 12$ $\quad x = 2, x = 4$

c) $y = 3\sin(2x)$ $\quad x = \frac{\pi}{2}, x = \frac{5\pi}{6}$

d) $y = 2\cos x$ $\quad x = \frac{\pi}{6}, x = \frac{\pi}{4}$

Q2) A curve is given by $y = 10 + 3x - x^2$

a) Determine where this curve intercepts the $x-$ axis

b) Hence calculate the area under the curve

Q3) The curve C has parametric equations $x = \mathrm{t}(1 + \mathrm{t})$ and $y = \frac{1}{1+t}$.

Find the exact area of the region bounded by C, the x-axis and the lines $x = 0$ and $x = 2$. $t \geq 0$

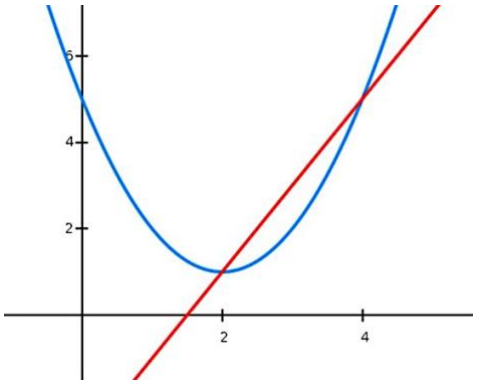
Q4) Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = a\cos\theta, \alpha = 0, \beta = \frac{\pi}{2}$

Finding the Area Between Two Curves

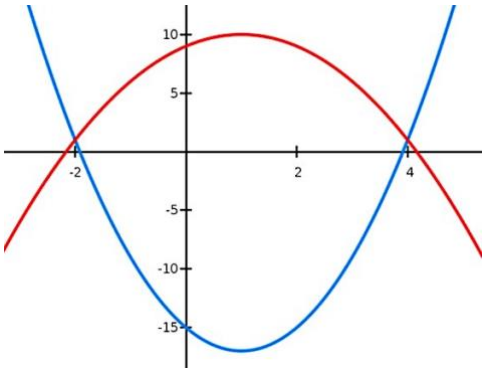
Q1) Find the area between the red line and the blue curve shown below

Red: $y = 2x - 3$, Blue: $y = x^2 - 4x + 5$



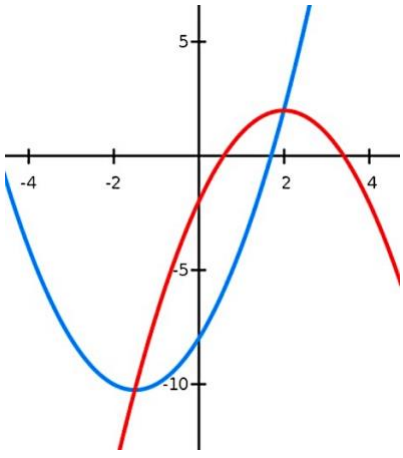
Q2) Find the area between the red and blue curves shown below

Red: $-x^2 + 2x + 9$, Blue: $2x^2 - 4x - 15$



Q3) Find the area between the red and blue curves shown below

Red: $y = -x^2 + 4x - 2$, Blue: $y = x^2 + 3x - 8$



Volumes of Revolution

Q1) Find the exact volume of the solid generated when each curve is rotated through 2π radians about the x-axis between the given limits.

a) $y = 10x^2$ between $x = 0$ and $x = 2$

b) $y = \sqrt{x}$ between $x = 2$ and $x = 10$

Q2) Find the exact volume of the solid generated when each curve is rotated through 2π radians about the x-axis between the given limits.

a) $y = \frac{2}{x+1}$ between $x = 0$ and $x = 2$.

b) $y = \sin 2x$ between $x = 0$ and $x = \frac{\pi}{2}$

Q3) Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y-axis between the given limits.

a) $y = \frac{1}{x}$ between $y = 1$ and $y = 3$

b) $y = 2e^{x^2}$ between $y = 2$ and $y = 4$

Q4) The curve C is given by parametric equations $x = e^t$ and $y = \sqrt{t-1}$.
The region bounded by the curve, the x-axis and the lines $x = e^2$ and $x = e^3$ is rotated through 2π radians about the x-axis. Find the volume of the solid of revolution formed.

Arc Length

Q1) Find the length of the arc on the given curve between the stated co-ordinates.

a) $y = \frac{2}{3}x^{\frac{3}{2}}$ between $x = 8$ and $x = 15$

b) $y^2 = \frac{4}{9}x^3$ between $y = 0$ and $y = 2\sqrt{3}$

c) $x = \ln \cos y$ between $y = 0$ and $y = \frac{\pi}{2}$

Q2) Find the arc length of the given curve on the indicated interval.

a) $x = t^2,\ y = 2t$ $\quad 0 \le t \le 2$

b) $x = 3 + 2\cos t,\ y = 2t + 2\sin t$ $\quad \frac{\pi}{3} \le t \le \pi$

Q3) A curve C has equation $r = a$.

Find the length of the arc of C on the interval $0 \leq \theta \leq 2\pi$.

Q4) A curve C has equation $r = 5e^{2\theta}$.

Find the length of the arc of C between the points on the curve with $\theta = 0$ & $\theta = \frac{\pi}{2}$.

Surface of Revolution

Q1) The arc of the curve with parametric equations $x = t^2,\ y = t - \frac{1}{3}t^3$ between the points $t = 0$ and $t = 1$ is rotated through 2π radians about the x-axis.

Calculate the area of the surface generated.

Q2) The curve with parametric equations $x = \sin^2 t\,,\ y = \cos^2 t\,, o \leq t \leq \frac{\pi}{2}$ is rotated through 2π radians about the y-axis to form a surface of revolution.

Find the exact area of the surface.

Q3) The graph with polar equation $r = \cos\theta$ with $0 \leq \theta \leq \pi$ is a full circle.

Find the area of the surface generated by rotating the circle about the line $\theta = \pm\frac{\pi}{2}$.

Q4) The curve with polar equation $r = e^{\theta}$ with $0 \leq \theta \leq \frac{\pi}{2}$ is rotated about the vertical line $\theta = \pm\frac{\pi}{2}$, forming a surface of revolution.

Find the exact area of the surface.