Differentiation Techniques

Solutions

The Power Rule

Q1) a) $f(x) = 4x^2$

$f'(x) = 8x$

b) $f(x) = 3x^3 + \frac{1}{2}$

$f'(x) = 9x^2$

c) $y = 5x^{-2}$

$\frac{dy}{dx} = -10x^{-3}$

d) $f(x) = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

e) $f(x) = \frac{x^4}{4} = \frac{1}{4}x^4$

$f'(x) = x^3$

f) $y = \frac{1}{2}x^{\frac{2}{3}}$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{1}{3}}$

Q2) a) $y = x^2 + 4$

$\frac{dy}{dx} = 2x$

b) $f(x) = \frac{1}{3}x^5 + 2x$

$f'(x) = \frac{5}{3}x^4 + 2$

c) $f(x) = \frac{1}{2}x^{-6} - 3x^2$

$f'(x) = -3x^{-7} - 6x$

d) $y = x^4 + 3x^{-3}$

$\frac{dy}{dx} = 4x^3 - 9x^{-4}$

e) $y = 2x^2 + 5x^2$

$\frac{dy}{dx} = 4x + 10x$

f) $h(x) = \frac{1}{3}x^{-3}$

$h'(x) = -x^{-4}$

Q3) a) $f(x) = \frac{1}{x^3}$

$f(x) = x^{-3}$

$f'(x) = -3x^{-4}$

b) $y = \frac{x^2 + 4}{3}$

$y = \frac{1}{3}x^2 + \frac{4}{3}$

$\frac{dy}{dx} = \frac{2}{3}x$

c) $y = 12\sqrt[3]{x}$

$y = 12x^{\frac{1}{3}}$

$\frac{dy}{dx} = 4x^{-\frac{2}{3}}$

d) $y = x^{\frac{-3}{4}}$

$\frac{dy}{dx} = -\frac{3}{4}x^{-\frac{7}{4}}$

e) $g(x) = \sqrt[4]{x}$

$g(x) = x^{\frac{1}{4}}$

$g'(x) = \frac{1}{4}x^{-\frac{3}{4}}$

f) $f(x) = 3\sqrt{x^3}$

$f(x) = 3x^{\frac{3}{2}}$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}}$

The Product & Quotient Rule

Q1) a) $f(x) = x(2x^2 + 3)$

$f'(x) = x(4x) + (2x^2 + 3)(1)$

$= 4x^2 + (2x^2 + 3)$

$= 6x^2 + 3$

b) $f(x) = x^3(4 - 4x)$

$f'(x) = x^3(-4) + (4 - 4x)(3x^2)$

$= -4x^3 + 12x^2 - 12x^3$

$= 12x^2 - 16x^3$

c) $f(x) = (3x + 5)(x^4 - 2)$

$f'(x) = (3x + 5)(4x^3) + (x^4 - 2)(3)$

$= 12x^4 + 20x^3 + 3x^4 - 6$

$= 15x^4 + 20x^3 - 6$

d) $y = \frac{1}{x^2}(x^3 + 3x) = x^{-2}(x^3 + 3x)$

$\frac{dy}{dx} = x^{-2}(3x^2 + 3) + (x^3 + 3x)(-2x^{-3})$

$= 3 + 3x^{-2} - 2 - 6x^{-2}$

$= 1 - 3x^{-2}$

e) $y = x^{-3}(2x^4 + 7)$

$\frac{dy}{dx} = x^{-3}(8x^3) + (2x^4 + 7)(-3x^{-4})$

$= 8 - 6 - 21x^{-4}$

$= 2 - 21x^{-4}$

f) $y = x^{-\frac{1}{2}}(5 - 4x^{-3})$

$\frac{dy}{dx} = x^{-\frac{1}{2}}(12x^{-4}) + (5 - 4x^{-3})(-\frac{1}{2}x^{-\frac{3}{2}})$

$= 12x^{-\frac{9}{2}} - \frac{5}{2}x^{-\frac{3}{2}} + 2x^{-\frac{9}{2}}$

$= 14x^{-\frac{9}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$

Q2) a) $f(x) = \frac{4x}{2x^3}$

$f'(x) = \frac{(4)(2x^3) - (6x^2)(4x)}{(2x^3)^2}$

$= \frac{8x^3 - 24x^3}{4x^6}$

$= -\frac{4}{x^3}$

b) $y = \frac{7x}{3x + 1}$

$\frac{dy}{dx} = \frac{(7)(3x + 1) - (3)(7x)}{(3x + 1)^2}$

$= \frac{21x + 7 - 21x}{(3x + 1)^2}$

$= \frac{7}{(3x + 1)^2}$

c) $y = \frac{x^2}{5 - 4x}$

$\frac{dy}{dx} = \frac{(2x)(5 - 4x) - (-4)(x^2)}{(5 - 4x)^2}$

$= \frac{10x - 8x^2 + 4x^2}{(5 - 4x)^2}$

$= \frac{10x - 4x^2}{(5 - 4x)^2}$

d) $f(x) = \frac{x^2 - 5x}{x^3}$

$f'(x) = \frac{(2x - 5)(x^3) - (3x^2)(x^2 - 5x)}{(x^3)^2}$

$= \frac{2x^4 - 5x^3 - 3x^4 + 15x^3}{x^6}$

$= \frac{10x^3 - x^4}{x^6} = \frac{10 - x}{x^3}$

e) $f(x) = \frac{2x^3 + 4}{x^2 - 3}$

$f'(x) = \frac{6x^2(x^2 - 3) - (2x)(2x^3 + 4)}{(x^2 - 3)^2}$

$= \frac{6x^4 - 18x^2 - 4x^4 - 8x}{(x^2 - 3)^2}$

$= \frac{2x^4 - 18x^2 - 8x}{(x^2 - 3)^2}$

f) $y = \frac{x^{-2} - 7}{x^{-3}}$

$\frac{dy}{dx} = \frac{(-2x^{-3})(x^{-3}) - (-3x^{-4})(x^{-2} - 7)}{(x^{-3})^2}$

$= \frac{-2x^{-6} + 3x^{-6} - 7x^{-4}}{x^{-6}}$

$= \frac{x^{-6} - 7x^{-4}}{x^{-6}}$

$= 1 - 7x^2$

Differentiating Trig Functions

Q1) a) $f(x) = 5\sin x$

$f'(x) = 5\cos x$

b) $f(x) = 3\sin 2x$

$f'(x) = 6\cos 2x$

c) $f(x) = 4\cos x$

$f'(x) = -4\sin x$

d) $f(x) = -2\cos(3x)$

$f'(x) = 6\sin(3x)$

e) $f(x) = \frac{1}{2}\cos 6x$

$f'(x) = -3\sin 6x$

f) $f(x) = -3\sin 2x$

$f'(x) = -6\cos 2x$

g) $f(x) = \frac{1}{3}\cos 9x$

$f'(x) = -3\sin 9x$

h) $f(x) = -4\sin\frac{x}{4}$

$f'(x) = -\cos\frac{x}{4}$