Applications

Questions

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The Equation of a Tangent to a Curve

Q1) Find the equation of the tangent line to the curve at the indicated point.

a) $y = x^2 + 3x + 6$ at $(1, 10)$

b) $y = x + \frac{1}{x}$ at $(2, \frac{5}{2})$

c) $y = 4\sqrt{x}$ at $(9, 12)$

d) $y = \cos x$ at $(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$

Increasing & Decreasing Functions

Q1) Is the given function increasing or decreasing at the indicated value?

a) $f(x) = x^3 - 2x^2 + 6x + 3$ when $x = 1$

b) $f(x) = 2x^3 + x^2 - 8x - 1$ when $x = -1$

c) $f(x) = x^4 - 2x^3 - 2x^2 - 3$ when $x = 2$

Q2) The function f is given by $f(x) = x^2 - \frac{3}{x}$. Is $f(x)$ increasing or decreasing when $x = -1$?

Determining Local Extrema of a Function

Q1) Find the co-ordinates of the stationary point(s) of the function.

a) $f(x) = 2x^4 - 12$

b) $f(x) = 4x$

c) $y = x + \frac{1}{x}$

d) $y = x - 3\sqrt{x}$

Q2) Find the stationary points of each of the following curves and determine their nature using a nature table.

a) $y = \frac{1}{3}x^3 - 2x^2 + 3x - 1$

b) $y = x^3 - 2x^2 - 4x + 1$

c) $y = 2 + 5x - x^2 - x^3$

d) $y = x^3(x - 2)$

Q3) Find the x co-ordinate of the stationary points of each of the following curves and determine their nature using the 2nd derivative.

a) $y = 4x^2 + 6x$

b) $y = x(x^2 - 4x - 3)$

Sketching Curves

Q1) Sketch the graph of the following functions by finding the x and y intercepts and the local maxima and minima.

a) $y = 4x^2 + 6x$

b) $y = x^3 - 3x^2$

Optimization

Q1) A storage container is in the shape of an open-topped, square-based cuboid and has a volume of 500 cm^3. The length of the container is x cm and the height is h cm.

a) Prove that the surface area of the container is given by the formula

$S = x^2 + \frac{2000}{x}$

b) Find the dimensions of the container which minimise this surface area.

Q2) A company is contracted to provide a service. The profit made by the company can be modelled by the following formula:

$P(x) = 192x - x^3 + 400$, where x is the number of jobs completed each day.

a) Determine the number of jobs that have to be completed daily to maximize profit.

b) Calculate the maximum profit made in one day.

Rolle's Theorem & The Mean Value Theorem

Q1) Use Rolle's Theorem to find all values of c in the given interval such that $f'(c) = 0$.

If you cannot use Rolle's Theorem, state why not.

a) $f(x) = x^2 - 3x$ $[0, 3]$

b) $f(x) = \frac{x^2 - 1}{x}$ $[-1, 1]$

c) $f(x) = \sin 2x$ $[\frac{\pi}{6}, \frac{\pi}{3}]$

Q2) Apply the Mean Value Theorem to f on the indicated interval to find all values of c such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

a) $f(x) = x^2$ $[-2, 1]$

b) $f(x) = \sqrt{x - 2}$ $[2, 6]$

c) $f(x) = \frac{x + 1}{x}$ $[\frac{1}{2}, 2]$

d) $f(x) = \sin x$ $[0, \pi]$