Trigonometry & Complex Numbers

Solutions

Working with Radians

Trigonometric Exact Values

Operations with Complex Numbers

Quadratic Equations

Working with Radians

Q1) a) $180° = \pi$ radians

b) $360° = 2\pi$ radians

c) $90° = \frac{\pi}{2}$ radians

d) $30° = \frac{\pi}{6}$ radians

e) $45° = \frac{\pi}{4}$ radians

f) $60° = \frac{\pi}{3}$ radians

Q2) a) $10° = 10\ x\ \frac{\pi}{180}$

$= 0.17$ radians

b) $40° = 40\ x\ \frac{\pi}{180}$

$= 0.7$ radians

c) $305° = 305\ x\ \frac{\pi}{180}$

$= 5.3$ radians

d) $24° = 24\ x\ \frac{\pi}{180}$

$= 0.42$ radians

Q3) a) $\frac{3\pi}{2} = 270°$

b) $\frac{3\pi}{4} = 135°$

c) $2\pi = 360°$

d) $2.5 \text{ rad} = 2.5\ x\ \frac{180}{\pi} = 143°$

e) $1.5 \text{ rad} = 1.5\ x\ \frac{180}{\pi} = 86°$

f) $3.5 \text{ rad} = 3.5\ x\ \frac{180}{\pi} = 201°$

Trigonometric Exact Values

Q1) a) $Sin\ 30° = \frac{1}{2}$

b) $Sin\ 45° = \frac{1}{\sqrt{2}}$

c) $Sin\ 60° = \frac{\sqrt{3}}{2}$

d) $Cos\ 30° = \frac{\sqrt{3}}{2}$

e) $Cos\ 45° = \frac{1}{\sqrt{2}}$

f) $Cos\ 60° = \frac{1}{2}$

g) $Tan\ 30° = \frac{1}{\sqrt{3}}$

h) $Tan\ 45° = \frac{1}{1}$

i) $Tan\ 60° = \frac{\sqrt{3}}{1}$

Q2) a) $Sin\ 225° = -Sin\ 45° = -\frac{1}{\sqrt{2}}$

b) $Cos\ 225° = -Cos\ 45° = \frac{1}{\sqrt{2}}$

c) $Cos\ 135° = -Sin\ 45° = -\frac{1}{\sqrt{2}}$

d) $Sin\ 135° = Sin\ 45° = \frac{1}{\sqrt{2}}$

Q3) a) $Sin\ \frac{\pi}{2} = 1$

b) $Cos\ \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

c) $Sin\ 2\pi = 0$

d) $Tan\ \frac{\pi}{3} = \sqrt{3}$

e) $Sin\ \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

f) $Cos\ \frac{\pi}{3} = \frac{1}{2}$

Q4) a) $Sin\ \frac{5\pi}{3} = -Sin\ \frac{\pi}{3} = -\frac{1}{2}$

b) $Sin\ \frac{3\pi}{2} = -1$

c) $Cos\ \frac{2\pi}{3} = -Cos\ \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

d) $Sin\ \frac{11\pi}{6} = -Sin\ \frac{\pi}{6} = -\frac{1}{2}$

e) $Cos\ \frac{3\pi}{4} = -Cos\ \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

f) $Tan\ \frac{5\pi}{6} = Tan\ \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Operations with Complex Numbers

Q2) a) $(5 + 2\text{i}) + (8 + 9\text{i})$

$= 13 + 11\text{i}$

b) $(-2 - 7\text{i}) + (1 + 3\text{i})$

$= -1 - 4\text{i}$

c) $5(4 + 3\text{i}) - 4(-1 + 2\text{i})$

$= 24 + 7\text{i}$

d) $(6 + 3i)(7 + 2i)$

$= 36 + 33i$

e) $(2 + 3i)^3$

$= (2 + 3i)(-5 + 12i)$

$= -10 + 36i^2 + 9i$

$= -46 + 9i$

f) $(4 + 5i)(4 - 5i)$

$= 16 + 20i - 20i - 25i^2$

$= 41$

Q2) a) $z - w$

$= (7 - 6i) - (7 + 6i)$

$= -12i$

b) $w + z$

$= (7 + 6i) + (7 - 6i)$

$= 14$

Q3) a) $z + z^*$

$= 6 - 5i + 6 + 5i$

$= 12$

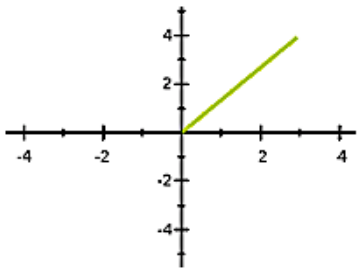
b) zz^*

$= (6 - 5i)(6 + 5i)$

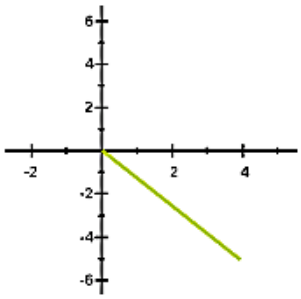
$= 36 - 25i^2 = 61$

Q4)

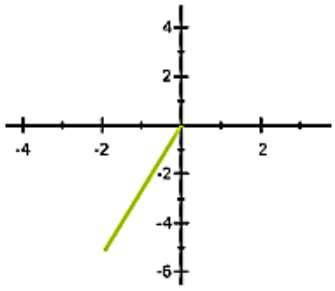
a) $z = 3 + 4i$



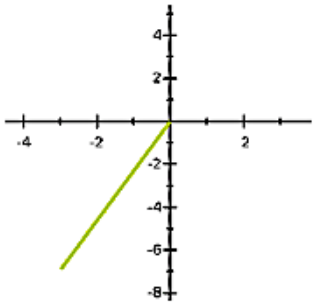
b) $z = 4 - 5i$



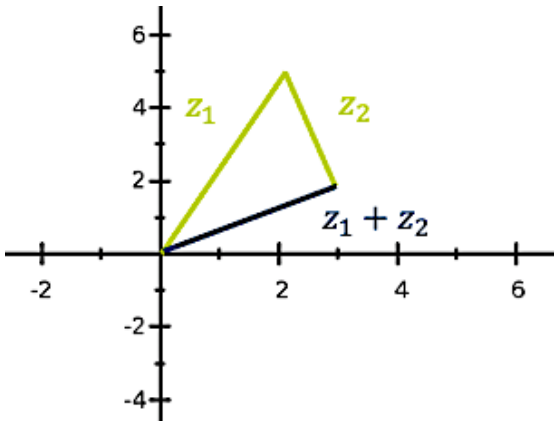
c) $z = -2 + 5i$



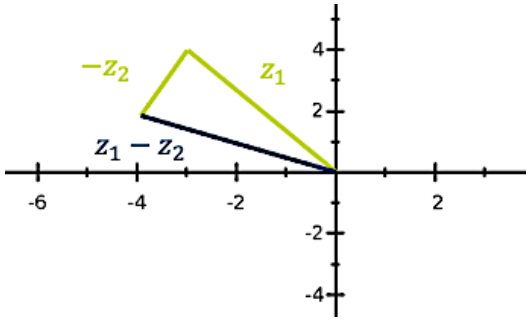
d) $z = -3 - 7i$



Q5) a)



b)



Quadratic Equations

Q1) a) $z^2 + 121 = 0$

$z^2 = -121$

$z = \sqrt{121}\sqrt{-1}$

$z = 11i$

b) $2z^2 + 120 = 0$

$2z^2 = -120$

$z^2 = -60$

$z = \sqrt{60}\sqrt{-1}$

$z = 2i\sqrt{15}$

c) $(z-3)^2 - 9 = -16$

$(z-3)^2 = -7$

$z - 3 = \sqrt{7}\sqrt{-1}$

$z = 3 \pm i\sqrt{7}$

d) $z^2 - 8z + 21 = 0$

$(z-4)^2 - 16 + 21 = 0$

$(z-4)^2 = -5$

$z - 4 = \sqrt{5}\sqrt{-1}$

$z = 4 \pm i\sqrt{5}$

Q2) a) The other root is the complex conjugate, $2 - 3i$

b) $\big(z - (2 + 3i)\big)\big(z - (2 - 3i)\big) = 0$

$z^2 - (2 - 3i)z - (2 + 3i)z + (2 + 3i)(2 - 3i) = 0$

$z^2 - 4z + 4 - 9i^2 = 0$

$z^2 - 4z + 13 = 0$

So, $p = -4$ and $q = 13$