Polynomials

Solutions

Polynomial Operations

Factorising Polynomials

Polynomial Graphs

Polynomial Operations

Q1) a) $f(x) + g(x) = (x^2 + 2x - 3) + (2x^2 - 4x - 6)$

$= 3x^2 - 2x - 9$

b) $f(x) - g(x) = (-3x^3 + 2x^2 + 7) - (2x^3 - 7x^2 + 4)$

$= -5x^3 + 9x^2 + 3$

Q2) a) $2f(x) = 2(3x^3 + 2x^2 - x + 5)$

$= 6x^3 + 4x^2 - 2x + 10$

b) $-3g(x) = -3(2x^2 - 3x - 3)$

$= -6x^2 + 9x + 9$

Q3) a) $(x + 2)\ x\ (x^3 + 4x^2 + 2x + 3)$

$= (x^4 + 4x^3 + 2x^2 + 3x) + (2x^3 + 8x^2 + 4x + 6)$

$= x^4 + 6x^3 + 10x^2 + 7x + 6$

b) $(2x - 1)\ x\ (x^3 + x^2 + 3x - 5)$

$= (2x^4 + 2x^3 + 6x^2 - 10x) + (-x^3 - x^2 - 3x + 5)$

$= 2x^4 + x^3 + 5x^2 - 13x + 5$

Factorising Polynomials

Q1) a) $2x^3 - 8x^2 + 16x = 2x(x^2 - 4x + 8)$

b) $x^3 + 4x^2 - 13x = x(x^2 + 4x - 13)$

c) $4x^4 - 12x^3 + 8x = x(4x^3 - 12x^2 + 8)$

d) $3x^3 + 3x^2 + 9 = 3(x^3 + x^2 + 3)$

Q2) a)

4	1	-5	2	8
		4	-4	-8
	1	-1	-2	0

Since the remainder is 0, $(x - 4)$ is a factor of $h(x)$.

$h(x) = (x - 4)(x^2 - x - 2)$

$= (x - 4)(x - 2)(x + 1)$

b)

-3	1	0	-13	-12
		-3	9	12
	1	-3	-4	0

Since the remainder is 0, $(x + 3)$ is a factor of $g(x)$.

$g(x) = (x + 3)(x^2 - 3x - 4)$

$= (x + 3)(x - 4)(x + 1)$

Q3 a)

3	1	-2	-9	18
		3	3	-18
	1	1	-6	0

Since the remainder is 0, $(x - 3)$ is a factor of $f(x)$.

b)

-2	1	5	2	-8	0
		-2	-6	8	0
	1	3	-4	0	0

Since the remainder is 0, $(x+2)$ is a factor of $f(x)$.

Q4) a) $x^3 - x$

Since there is a common factor of $'x'$ there is no need to use synthetic division.

$x^3 - x = x(x^2 - 1)$

$= x(x-1)(x+1)$

b) $x^3 - 13x - 12$

Trying the factors of -12 in synthetic division gives a zero remainder for $x = 3$.

3	1	0	-13	12
		3	9	-12
	1	3	-4	0

$x^3 - 13x - 12 = (x-3)(x^2 + 3x - 4)$

$= (x-3)(x+4)(x-1)$

c) $x^3 - x^2 - x + 1$

Trying the factors of 1 in synthetic division gives a zero remainder for $x = 1$.

1	1	-1	-1	1
		1	0	-1
	1	0	-1	0

$x^3 - x^2 - x + 1 = (x-1)(x^2 - 1)$

$= (x-1)(x-1)(x+1)$

$= (x-1)^2(x+1)$

d) $x^3 - 4x^2 - 3x + 18$

Trying the factors of 18 in synthetic division gives a zero remainder for $x = 3$.

3	1	-4	-3	18
		3	-3	-18
	1	-1	-6	0

$x^3 - 4x^2 - 3x + 18 = (x - 3)(x^2 - x - 6)$

$= (x - 3)(x - 3)(x + 2)$

Q5) a) $x^4 - 2x^2 + 1$

Trying the factors of 1 in synthetic division gives a zero remainder for $x = 1$.

1	1	0	-2	0	1
		1	1	-1	-1
	1	1	-1	-1	0

$x^4 - 2x^2 + 1 = (x - 1)(x^3 + x^2 - x - 1)$

We now have to factorise $x^3 + x^2 - x - 1$ so we use synthetic division again.

Trying the factors of -1 in synthetic division gives a zero remainder for $x = 1$.

1	1	1	-1	-1
		1	2	1
	1	2	1	0

So, $x^3 + x^2 - x - 1 = (x - 1)(x^2 + 2x + 1)$

$= (x - 1)(x + 1)(x + 1)$

$= (x - 1)(x + 1)^2$

So, $x^4 - 2x^2 + 1 = (x - 1)(x - 1)(x + 1)^2$

$= (x - 1)^2(x + 1)^2$

b) $x^4 - 3x^3 - 6x^2 + 8x$

$= x(x^3 - 3x^2 - 6x + 8)$

Factorise $x^3 - 3x^2 - 6x + 8$ using synthetic division. Trying the factors of 8 in synthetic division gives a zero remainder for $x = 1$.

1	1	-3	-6	8
		1	-2	-8
	1	-2	-8	0

$x^3 - 3x^2 - 6x + 8 = (x - 1)(x^2 - 2x - 8)$

$= (x - 1)(x - 4)(x + 2)$

So, $x^4 - 3x^3 - 6x^2 + 8x = x(x - 1)(x - 4)(x + 2)$

Polynomial Graphs

Q1) a) Since the roots are at $x = -1, x = 2, x = 3$ the function has the following factors:

$(x - 3), (x - 2), (x + 1)$

So, $f(x) = (x - 3)(x - 2)(x + 1)$

$= x^3 - 4x^2 + x + 6$

b) Since the roots are at $x = 2, x = 2, x = -4$ the function has the following factors:

$(x - 2), (x - 2), (x + 4)$

So, $f(x) = (x - 2)(x - 2)(x + 4)$

$= x^3 - 12x + 16$

c) Since the roots are at $x = -2, x = -1, x = 2, x = 4$, the function has these factors:

$(x - 4), (x - 2), (x + 1), (x + 2)$

So, $f(x) = (x - 4)(x - 2)(x + 1)(x + 2)$

$= x^4 - 3x^3 - 8x^2 + 12x + 16$

d) Since the roots are at $x = -3, x = -3, x = 1, x = 5$ the function has these factors:

$(x - 5), (x - 1), (x + 3), (x + 3)$

So, $f(x) = (x - 4)(x - 2)(x + 1)(x + 2)$

$= x^4 - 3x^3 - 8x^2 + 12x + 16$

Q2) a) The roots of the graph are at $x = -4, x = -2, x = 2$ so the equation is

$f(x) = (x - 2)(x + 2)(x + 4)$

$= x^3 + 4x^2 - 4x - 16$

b) The roots of the graph are at $x = -3, x = 1$ so the equation is

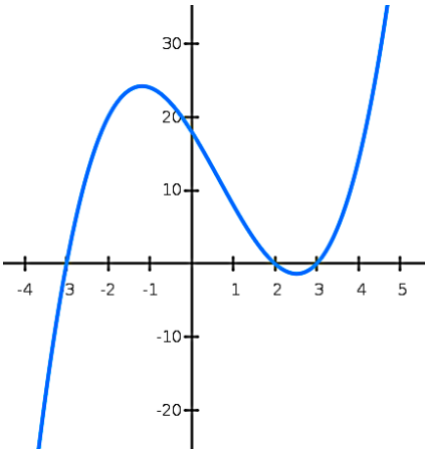
$f(x) = (x - 1)(x - 1)(x + 3)$

$= (x - 1)^2(x + 3)$

Q3) a) $f(x) = (x + 3)(x - 2)(x - 3)$

So, the graph has roots at $x = -3, x = 2, x = 3$

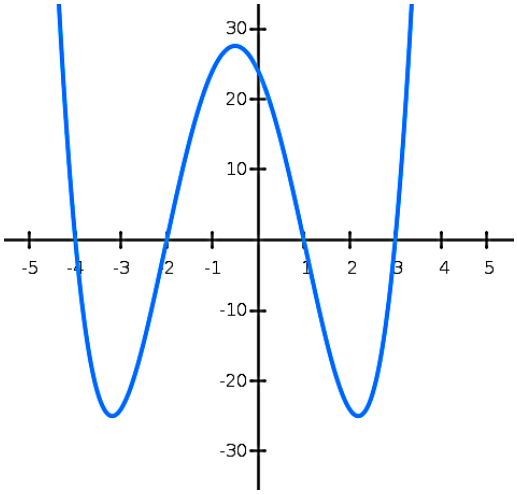
Mark these on an x/y axis and fill in the shape of a cubic function.



b) $f(x) = (x - 3)(x - 1)(x + 2)(x + 4)$

So, the graph has roots at $x = -4, x = -2, x = 1, x = 3$

Mark these on an x/y axis and fill in the shape of a quartic function.



c) $f(x) = (x-2)^2(x+1)$

So, the graph has roots at $x = -1, x = 2$

$x = 2$ is a repeated root which means that this is also a turning point on the graph of $f(x)$. Mark these on an x/y axis and fill in the shape of a cubic function.