Functions & Graphs

Solutions

Functions

Straight Line Graphs

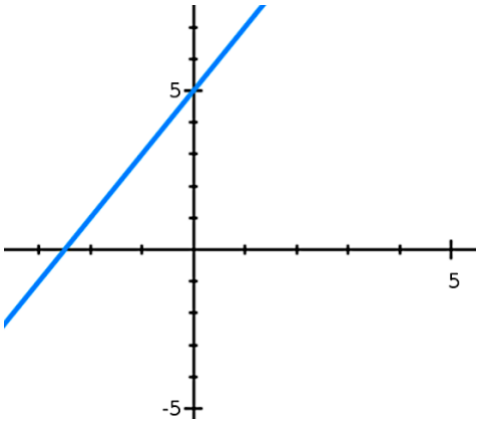
Quadratic Functions

Functions

Q1) a) $f(x) = 2x + 5$

Input values, x	-3	-2	-1	0	1	2	3
Output values, $f(x)$	-1	1	3	5	7	9	11

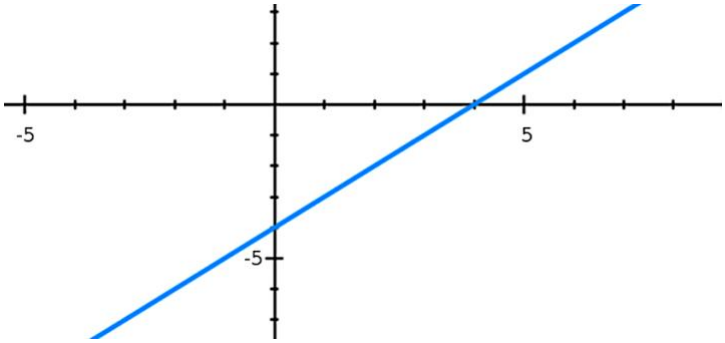
$(-3, -1), (-2, 1), (-1, 3), (0, 5), (1, 7), (2, 9), (3, 11)$



b) $f(x) = x - 4$

Input values, x	-3	-2	-1	0	2	5	8
Output values, $f(x)$	-7	-6	-5	-4	-2	1	4

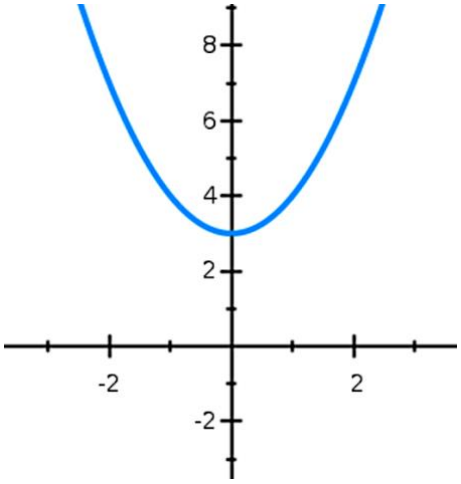
$(-3,-7),(-2,-6),(-1,-5),(0,-4),(2,-2),(5,1),(8,4)$



c) $f(x) = x^2 + 3$

Input values, x	-3	-2	-1	0	1	2	3
Output values, $f(x)$	12	7	4	3	4	7	12

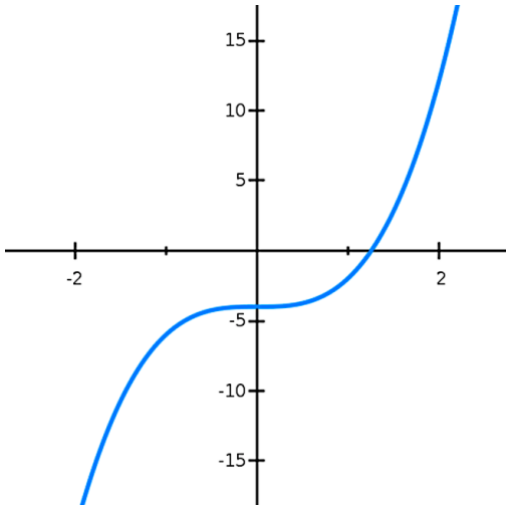
$(-3,12),(-2,7),(-1,4),(0,3),(1,4),(2,7),(3,12)$



d) $f(x) = 2x^3 - 4$

Input values, x	-3	-2	-1	0	1	2	3
Output values, $f(x)$	-50	-20	-6	-4	-2	12	50

$(-3,-50),(-2,-20),(-1,-6),(0,4),(1,-2),(2,12),(3,50)$



Q2) a) No maximum or minimum as straight lines continue indefinitely.

b) No maximum or minimum as straight lines continue indefinitely.

c) Minimum value is 3. There is no maximum value.

d) No maximum or minimum as the graph continues indefinitely.

Q3) a) $f(x) = 2x + 3$

$f(5) = 13$

$f(1) = 5$

Rate of change $= \frac{13-5}{5-1} = \frac{8}{4} = 2$

b) $f(x) = x^2 - 2$

$f(-2) = 2$

$f(-6) = 34$

Rate of change $= \frac{2-34}{-2-(-6)} = \frac{-32}{4} = -8$

c) $f(x) = x^3$ on the interval $1 \leq x \leq 4$

$f(4) = 64$

$f(1) = 1$

Rate of change $= \frac{64-1}{4-1} = \frac{63}{3} = 21$

Straight Line Graphs

Q1) a) $A = (2,5),\ B = (3,7)$

$m_{AB} = \frac{7-5}{3-1} = \frac{2}{2} = 1$

b) $A = (-3,-5),\ B = (4,7)$

$m_{AB} = \frac{7-(-5)}{4-(-3)} = \frac{12}{7}$

Q2) a) $y - b = m(x - a)$

$y - 3 = 4(x - 1)$

$y - 3 = 4x - 4$

$y = 4x - 1$

b) $y - b = m(x - a)$

$y - 4 = -1(x - (-1))$

$y - 4 = -1(x + 1)$

$y - 4 = -x - 1$

$y = -x + 3$

Q3) a) $C = (2,-4), D = (3,7)$

$m_{CD} = \frac{7-(-4)}{3-2} = \frac{11}{1} = 11$

Using $y - b = m(x - a)$ with $(3,7)$ gives

$y - 7 = 11(x - 3)$

$y - 7 = 11x - 33$

$y = 11x - 26$

b) $R = (2,-3),\ S = (4,-2)$

$m_{RS} = \frac{-2-(-3)}{4-2} = \frac{1}{2}$

Using $y - b = m(x - a)$ with $(2,-3)$ gives

$y - (-3) = \frac{1}{2}(x - (-3))$

$y + 3 = \frac{1}{2}x + \frac{3}{2}$

$y = \frac{1}{2}x - \frac{3}{2}$

Q4) a) $y = 3x - 5$

y-intercept $= (0,-5)$

x-intercept $= (\frac{5}{3}, 0)$

b) $y = -2x + 7$

y-intercept $= (0,7)$

x-intercept $= (\frac{7}{2}, 0)$

c) $2y = 4x + 6$

$y = 2x + 3$

y-intercept $= (0, 3)$

x-intercept $= (-\frac{3}{2}, 0)$

d) $\frac{1}{2}y = -3x - 5$

$y = -6x - 10$

y-intercept $= (0, -10)$

x-intercept $= (-\frac{5}{3}, 0)$

Q5) a) Select two points on the line. I'll go for $(2, 0)$ and $(0, -4)$.

Gradient $= \frac{-4 - 0}{0 - 2} = \frac{-4}{-2} = 2$

From the graph, the y −intercept is -4

So, the equation is $y = 2x - 4$

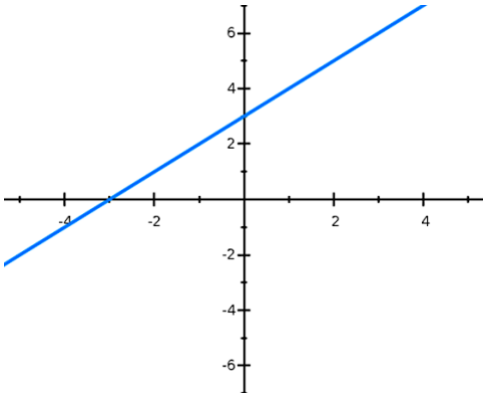
b) Select two points on the line. I'll go for $(0, -5)$ and $(-1, -2)$.

Gradient $= \frac{-2 - (-5)}{-1 - 0} = \frac{3}{-1} = -3$

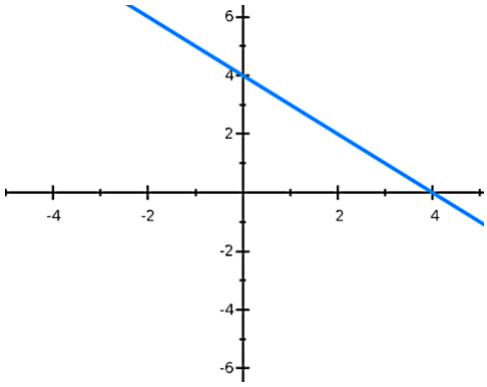
From the graph, the y −intercept is -5

So, the equation is $y = -3x - 5$

Q6) a) $y = x + 3$



b) $y = -x + 4$



Quadratic Functions

Q1) a) $x^2 + 10x + 25 = 0$

$(x + 5)(x + 5) = 0$

$x = -5$

b) $x^2 - 10x + 16 = 0$

$(x - 8)(x - 2) = 0$

$x = 8,\ x = 2$

c) $x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0,\ x = 3$

d) $x^2 - 49 = 0$

$(x - 7)(x + 7) = 0$

$x = 7,\ x = -7$

e) $x^2 + 7 + 5 = 0$

$\mathrm{a} = 1, \mathrm{b} = 7, \mathrm{c} = 5$

$x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$

$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 5}}{2 \times 1}$

$x = \frac{-7 \pm \sqrt{49 - 20}}{2}$

$x = \frac{-7 \pm \sqrt{29}}{2}$

$x = -0.81$ and $x = -6.19$

f) $x^2 + 2x - 3 = 0$

$a = 1, b = 2, c = -3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-3)}}{2 \times 1}$

$x = \frac{4 \pm \sqrt{16}}{2}$

$x = 4$ and $x = 0$

Q2) a) $x^2 - 3x - 3 = 0$

$a = 1, b = -3, c = -3$

$b^2 - 4ac = (-3)^2 - 4 \cdot 1 \cdot (-3)$

$= 21$

$b^2 - 4ac > 0$, two real roots.

b) $x^2 + 5x - 7 = 0$

$a = 1, b = 5, c = -7$

$b^2 - 4ac = 5^2 - 4 \cdot 1 \cdot (-7)$

$= 53$

$b^2 - 4ac > 0$, two real roots.

c) $2x^2 + 5 = 0$

$a = 2, b = 0, c = 5$

$b^2 - 4ac = -4 \cdot 2 \cdot 5$

$= -40$

$b^2 - 4ac < 0$, no real roots.

d) $x^2 + 5x = 0$

$a = 1, b = 5, c = 0$

$b^2 - 4ac = 5^2 - 4 \cdot 1 \cdot 0$

$= 25$

$b^2 - 4ac > 0$, two real roots.

Q3) a) $y = (x-3)^2 + 1$

Turning point $= (3, 1)$.

b) $y = (x-4)^2 - 5$

Turning point $= (4, -5)$.

c) $y = -(x-2)^2 - 2$

Turning point $= (2, -2)$.

d) $y = (x+6)^2$

Turning point $= (-6, 0)$.

Q4) a) $y = x^2 - 8x + 3$

$= (x-4)^2 - 13$

Turning point is at $(4, -13)$.

b) $y = x^2 + 6x + 17$

$= (x+3)^2 + 8$

Turning point is at $(-3, 8)$.

c) $y = x^2 + 10x$

$= (x+5)^2 - 25$

Turning point is at $(-5, -25)$.

d) $y = x^2 + 4x + 12$

$= (x+2)^2 + 8$

Turning point is at $(-2, 8)$.