Algebra & Equations

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Algebraic Expressions

Q1) a) $P = 2r - 2s$

Let $r = -4$ and $s = -3$

$P = (2\,x - 4) - (2\,x - 3)$

$= -8 - (-6)$

$= -8 + 6 = -2$

b) $Q = 3(2r - 3s)$

Let $r = 1$ and $s = -1$

$Q = 3[(2\,x\,1) - (3\,x - 1)]$

$= 3[2 - (-3)]$

$= 3\,x\,5 = 15$

Q2) a) $3a - x = a + 2x$

$3a - a = 2x + x$

$2a = 3x$

$a = \frac{3x}{2}$

b) $a + 2 = x(3 + a)$

$a + 2 = 3x + ax$

$a - ax = 3x - 2$

$a(1 - x) = 3x - 2$

$a = \frac{3x - 2}{1 - x}$

c) $r = \frac{\sqrt{t}}{5}$

$\sqrt{t} = 5r$

$t = (5r)^2 = 25r^2$

d) $\sqrt{t + 3} = s$

$t + 3 = s^2$

$t = s^2 - 3$

e) $z = \frac{a - 3}{5 - a}$

$z(5 - a) = a - 3$

$5z - az = a - 3$

$5z + 3 = a(1 + z)$

$a(1 + z) = 5z + 3$

$a = \frac{5z + 3}{1 + z}$

f) $x(a-1) = b(a+2)$

$xa - x = ba + 2b$

$xa - ba = 2b + x$

$a(x-b) = 2b + x$

$a = \frac{2b+x}{x-b}$

Q3) a) $5(x-4)+7$

$= 5x - 20 + 7$

$= 5x - 13$

b) $2 + 3(2-t)$

$= 2 + 6 - 2t$

$= 8 - 2t$

c) $-3(c-2) + c$

$= -3c + 6 + c$

$= 6 - 2c$

d) $5x + 2(x-3)$

$= 5x + 2x - 3$

$= 7x - 3$

e) $4(a-5) + 3(2a+1)$

$= 4a - 20 + 6a + 3$

$= 10a - 17$

f) $m(m-2) - m(3m+5)$

$= m^2 - 2m - 3m^2 - 5m$

$= -2m^2 - 5m$

Q4) a) $x^2 + 4x = x(x+4)$

b) $x^2 - 49 = (x-7)(x+7)$

c) $2x - x^2 = x(2-x)$

d) $x^2 + 7x = x(x+7)$

e) $x^2 - 8x + 15 = (x-3)(x-5)$

f) $36 - r^2 = (6-r)(6+r)$

g) $x^2 + 3x - 18 = (x+6)(x-3)$

h) $4x^2 - 64 = (2x-8)(2x+8)$

Q5) a) $x^2 - 4x + 7 = (x-2)^2 + 3$

b) $x^2 + 4x - 23 = (x+2)^2 - 27$

c) $x^2 - 16x + 2 = (x-8)^2 - 62$

d) $x^2 + 8x = (x+4)^2 - 16$

Algebraic Fractions

Q1) a) $\frac{4t-6}{2} = \frac{2(t-3)}{2} = t-3$

b) $\frac{2x+3x}{x} = \frac{5x}{x} = 5$

c) $\frac{12}{3r-9} = \frac{12}{3(r-3)} = \frac{4}{r-3}$

d) $\frac{a^2-3a}{a^2+6a} = \frac{a(a-3)}{a(a+6)} = \frac{a-3}{a+6}$

e) $\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$

f) $\frac{t^2-2t}{t^2+t-6} = \frac{t(t-2)}{(t+3)(t-2)} = \frac{t}{t+3}$

Q2) a) $\frac{3a}{5} \times \frac{10}{7a} = \frac{30a}{35a} = \frac{6}{7}$

b) $\frac{2}{5x} \times \frac{4x}{7} = \frac{8x}{35x} = \frac{8}{35}$

c) $\frac{5x}{6} \div \frac{7x}{8} = \frac{5x}{6} \times \frac{8}{7x} = \frac{20}{21}$

d) $\frac{2x}{5y} \div \frac{8}{15y} = \frac{2x}{5y} \times \frac{15y}{8} = \frac{3x}{4}$

Q3) a) $\frac{(x+3)}{4} + \frac{(x-2)}{3}$

$= \frac{3(x+3)}{12} + \frac{4(x-2)}{12}$

$= \frac{3x+9+4x-8}{12}$

$= \frac{7x+1}{12}$

b) $\frac{2}{(x+3)} - \frac{3}{(x-2)}$

$= \frac{2(x-2)}{(x+3)(x+2)} - \frac{3(x+3)}{(x+3)(x+2)}$

$= \frac{2x-4-3x-9}{(x+3)(x+2)}$

$= \frac{-x-13}{(x+3)(x+2)}$

c) $\frac{3}{x} + \frac{2}{(x-3)}$

$= \frac{3(x-3)}{x(x-3)} + \frac{2x}{x(x-3)}$

$= \frac{3x-9+2x}{x(x-3)}$

$= \frac{x-9}{x(x-3)}$

d) $\frac{2}{(x-1)} - \frac{1}{(x+1)}$

$= \frac{2(x+1)}{(x-1)(x+1)} - \frac{x-1}{(x-1)(x+1)}$

$= \frac{2x+2-x+1}{(x-1)(x+1)}$

$= \frac{x+3}{(x-1)(x+1)}$

Equations & Inequations

Q1) a) $3x = 20 - x$

$4x = 20$

$x = 5$

b) $t + 3 = 5 - t$

$2t = 2$

$t = 1$

c) $3 + 5s = 2s + 13$

$3s = 10$

$s = \frac{10}{3}$

d) $5x - 7 = 3x$

$2x = 7$

$x = \frac{7}{2}$

e) $x = 3x - 2 + 7$

$-5 = 2x$

$x = -\frac{5}{2}$

f) $4a = 3 - 2a - 23$

$6a = -20$

$a = -\frac{20}{6} = -\frac{10}{3}$

g) $2(x + 3) = -2(x + 4)$

$2x + 6 = -2x - 8$

$4x = -14$

$x = -\frac{14}{4}$

$x = -\frac{7}{2}$

h) $3r - 7(1 + r) = 12$

$3r - 7 - 7r = 12$

$-4r = 19$

$r = -\frac{19}{4}$

Q2) a) $3n > 9$

$n > 3$

b) $t + 2 < -1$

$t < -3$

c) $7k > 3k - 16$

$4k > -16$

$k > -4$

f) $8 + 2x > 3(4 - x)$

$8 + 2x > 12 - 3x$

$x > \frac{4}{5}$

Sequences

Q1) a) i) $u_n = 5n + 2$

$u_1 = 5(1) + 2 = 7$

$u_2 = 5(2) + 2 = 12$

$u_3 = 5(3) + 2 = 17$

$u_4 = 5(4) + 2 = 22$

ii) $a = 7$ and $d = 5$

b) i) $u_n = 9 - 2n$

$u_1 = 9 - 2(1) = 7$

$u_2 = 9 - 2(2) = 5$

$u_3 = 9 - 2(3) = 3$

$u_4 = 9 - 2(4) = 1$

ii) $a = 7$ and $d = -2$

Q2) $d = 3$

Let $n = 10$ to give $a_{10} = 2 + (10 - 1)3$

$= 2 + 27$

$= 29$

Let $n = 15$ to give $a_{15} = 2 + (15 - 1)3$

$= 2 + 42$

$= 44$

Q3) $x, 3x, 5x, \ldots, 35x$

$a = x$ and $d = 2x$

Then $u_n = x + (n - 1)2x$

Set this equal to the final term in the sequence:

$x + 2nx - 2x = 35x$

$2nx = 36x$

$2n = 36$

$n = 18$

So, there are 18 terms in the sequence.

Q4) $a = 14$ and $u_4 = 32$

$u_n = 14 + (n-1)d$

Set this equal to $u_4 = 32$:

$14 + \big((4) - 1\big)d = 32$

$14 + 3d = 32$

$3d = 18$

$d = 6$

Q5) a) $5, 15, 45, ..$

$a = 5$ and $r = 3$

$u_4 = 5(3)^3 = 135$

$u_5 = 5(3)^4 = 405$

$u_6 = 5(3)^5 = 1215$

b) $1, p, p^2, ..$

$a = 1\ and\ r = p$

$u_4 = 1(p)^3 = p^3$

$u_5 = 1(p)^4 = p^4$

$u_6 = 1(p)^5 = p^5$

Q6) $3, 6, 12, 24, 48, ..$

$a = 3, r = 2$

Let $n = 8$, to give $a_8 = 3\ x\ 2^7$

$= 3\ x\ 128$

$= 384$

Let $n = 12$, to give $a_{12} = 3\ x\ 2^{11}$

$= 3\ x\ 2{,}048$

$= 6{,}144$

Q7) a) $\frac{u_2}{u_1} = \frac{x}{3}$ and $\frac{u_3}{u_2} = \frac{9}{x}$

$\frac{u_2}{u_1} = \mathrm{r} = \frac{u_3}{u_2}$ giving

$\frac{x}{3} = \frac{9}{x}$

$x^2 = 27$

$x = 3\sqrt{3}$

b) $r = \frac{u_2}{u_1} = \frac{3\sqrt{3}}{3}$

$r = \sqrt{3}$

$u_4 = 3\big(\sqrt{3}\big)^3$

$u_4 = 9\sqrt{3}$

Q8) $ar^5 = 32$ (1)

$ar^2 = 4$ (2)

$\frac{ar^5}{ar^2} = \frac{32}{4} = 8 \Longrightarrow r^3 = 8 \Longrightarrow r = 2$

Substituting $r = 2$ into (2) gives

$a(2)^2 = 4 \Longrightarrow a = 1$