2023 National 5 Mathematics Paper 1

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Question 1, (2)

$$2\frac{1}{6} \div \frac{8}{9} = \frac{13}{6} x \frac{9}{8}$$

$$= \frac{13}{2} x \frac{3}{8}$$

$$= \frac{39}{16}$$

$$= 2\frac{7}{16}$$

Question 2, (3)

$(x + 7)^2 + 6(x^2 - 10)$

$= (x + 7)(x + 7) + 6(x^2 - 10)$

$= x^2 + 7x + 7x + 49 + 6x^2 - 60$

$= 7x^2 + 14x - 11$

Question 3, (3)

$2x + 3y = 8$ (1)

$5x + 2y = -2$ (2)

Multiply (1) by 2 and multiply (2) by 3 to give

$4x + 6y = 16$ (3)

$15x + 6y = -6$ (4)

(4) − (3) gives

$11x = -22$

$x = -2$

Substitute $x = -2$ into (1) to give

$2(-2) + 3y = 8$

$-4 + 3y = 8$

$3y = 12$

$y = 4$

Question 4, (1) (1) (1)

a) i) $a = -3$

ii) $b = 2$

b) $y = (x - 3)^2 + 2$

Let $x = 0$ to give

$y = (0 - 3)^2 + 2$

$y = 9 + 2$

$y = 11$

So, $c = 11$

Question 5, (2)

$f(x) = 4x^2 + 6x - 1$

$a = 4, b = 6, c = -1$

$b^2 - 4ac = 6^2 - 4(4)(-1)$

$= 36 + 16$

$= 42$

Since $b^2 - 4ac > 0$ there are two real and distinct roots.

Question 6, (3)

Using the Cosine rule gives

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 6^2 + 5^2 - 2(6)(5)(\frac{1}{5})$

$c^2 = 36 + 25 - 12$

$c^2 = 49$

$c = 7\ m$

Question 7, (3) (1)

a) The points given on the line are $(5, 20000)$ and $(25, 50000)$.

$$m = \frac{50000 - 20000}{25 - 5}$$

$$m = \frac{30000}{20}$$

$$m = \frac{3000}{2} = 1500$$

Using $y - b = m(x - a)$ with $(5, 20000)$ gives

$$y - 20000 = 1500(x - 5)$$

$$y - 20000 = 1500x - 7500$$

$$y = 1500x + 125000$$

Rewrite with P and T to give

$$P = 1500T + 125000$$

b) Substitute $T = 8$ to give

$$P = 1500(8) + 125000$$

$$P = 24{,}500$$

So, £24,500.

Question 8, (2)

$$\frac{12}{\sqrt{15}} = \frac{12}{\sqrt{15}} \, x \, \frac{\sqrt{15}}{\sqrt{15}}$$

$$= \frac{12\sqrt{15}}{15}$$

$$= \frac{4\sqrt{15}}{5}$$

Question 9, (3) (2)

a) Start by ordering the numbers to give

31 33 35 36 38 41 41 42 47 55

Median $= Q_2 = \frac{38 + 41}{2} = 39.5$

$Q_1 = 35, Q_3 = 42$

Interquartile Range $= Q_3 - Q_1 = 42 - 35 = 7$

b) On average the magazine readers are younger than the newspaper readers.

The ages of the magazine readers is more consistent than the newspaper readers since the interquartile range is lower for the magazine readers.

Question 10, (4)

Consider a right angled triangle from the midpoint of AB to A to C.

This triangle has a short side of 30 cm, and a long side of 50 cm. Let the other short side be a. Then, using Pythagoras we have,

$a^2 + 30^2 = 50^2$

$a^2 + 900 = 2500$

$a^2 = 1600$

$a = 40\ cm$

So, the width $= 40 + radius$

$= 40 + 50$

$= 90\ cm$

Question 11, (1)

Either use the CAST diagram or the symmetry of the Sine Graph to answer this question.

$\sin 330° = -\sin 30° = -0.5$

Question 12, (3)

$$\frac{5c^{-2}}{c^3\ x\ c^4} = \frac{5c^{-2}}{c^7}$$

$$= 5c^{-9}$$

$$= \frac{5}{c^9}$$

Question 13, (1) (1)

a) $a = 210$

b) $b = 1$

Question 14, (3)

$\frac{x+1}{3} - 2 > \frac{3x}{5}$

Multiply by 3 to give

$x + 1 - 6 > \frac{9x}{5}$

$x - 5 > \frac{9x}{5}$

Multiply by 5 to give

$5x - 25 > 9x$

$-25 > 4x$

$\frac{-25}{4} > x$

$x < \frac{-25}{4}$

2023 National 5 Mathematics Paper 2

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Question 1, (3)

An 11% depreciation can be expressed as 0.89 i.e. $100 - 11 = 89\%$.

$20{,}000\ x\ 0.89 = 17{,}800$

A 6% depreciation can be expressed as 0.94 i.e. $100 - 6 = 94\%$

$17{,}800\ x\ 0.94^2 = 15,728.08$

So, the value is £15,728.08

Question 2, (3)

$300 \div (6.64\ x\ 10^{-24})$

$= 4.518\ x\ 10^{25}$

$= 4.52\ x\ 10^{25}$ to 3 significant figures.

Question 3, (3)

Arc Length $= \frac{106}{360}\ x\ \pi\ x\ 18.3$

$= 16.93\ m$

Question 4, (3)

Using the Sine Rule gives

$$\frac{\sin J}{j} = \frac{\sin K}{k}$$

$$\frac{\sin 25}{7} = \frac{\sin K}{10}$$

$$\sin K = \frac{10 \sin 25}{7}$$

$\sin K = 0.60374\ldots$

$K = sin^{-1}(0.60374\ldots)$

$K = 37.1°$

$JKL = 37.1°$

Question 5, (2)

Angles at the centre $= 360 \div 10 = 36°$

Other angles in each triangle $= \frac{180 - 36}{2} = 72°$

Using the straight line gives $180 - 72 - 72 = 36°$

Shaded area angle $= 90 + 36 = 126°$

Question 6, (3)

$94,500 = 108\%$

Divide both sides by 108 to give

$875 = 1\%$

Multiply both sides by 100 to give

$87{,}500 = 100\%$

So, Nadim paid £87,500 for the flat.

Question 7, (3)

$P = \frac{1}{3}mn - r$

$P + r = \frac{1}{3}mn$

$3(P + r) = mn$

$m = \frac{3(P + r)}{n}$

Question 8, (4)

The wall is perpendicular (i.e. right-angled) to the ground if the triangle ABC is right-angled.

$4^2 + 7^2 = 16 + 49 = 65$

$8^2 = 64$

Since $65 \neq 64$, by the converse of Pythagoras Theorem, the triangle is not right-angled.

Therefore, the wall is not perpendicular to the ground.

Question 9, (4)

The block is a pyramid with a smaller pyramid removed from the top.

Volume of large pyramid $= \frac{1}{3}\ x\ 90^2\ x\ 108$

$= 291{,}600\ cm^3$

Volume of small pyramid $= \frac{1}{3}\ x\ 40^2\ x\ 48$

$= 25{,}600\ cm^3$

Volume of block $= 291{,}600 - 25{,}600 = 266{,}000\ cm^3$

Question 10, (3)

$$\frac{7}{x-3} - \frac{2}{x} = \frac{7x}{x(x-3)} - \frac{2(x-3)}{x(x-3)}$$

$$= \frac{7x - 2(x-3)}{x(x-3)}$$

$$= \frac{7x - 2x + 6)}{x(x-3)}$$

$$= \frac{5x + 6}{x(x-3)}$$

Question 11, (4)

$h = 20\cos x + 147$

Substitute $h = 150$ to give

$150 = 20\cos x + 147$

$20\cos x = 3$

$\cos x = \frac{3}{20}$

$cos^{-1}\left(\frac{3}{20}\right) = 81.4°$

From CAST $x = 81.4°$ and $x = 360 - 81.4 = 278.6°$

Question 12, (3)

$$\frac{x^2 - 16}{x^2 + x - 20}$$
$$= \frac{(x-4)(x+4)}{(x+5)(x-4)}$$
$$= \frac{(x+4)}{(x+5)}$$

Question 13, (2)

$$2sin^2x + 2cos^2x = 2(sin^2x + cos^2x)$$
$$= 2(1)$$
$$= 2$$

Question 14, (2) (4)

a) Volume $= l\ x\ b\ x\ h$

$$= (x+7)(x)(2)$$
$$= 2x(x+7)$$
$$= 2x^2 + 14x$$

But since the volume is 45 we have

$2x^2 + 14x = 45$

$2x^2 + 14x - 45 = 0$

b) We need to solve $2x^2 + 14x - 45 = 0$

Since this doesn't factorise we have to use the quadratic formula

$a = 2, b = 14, c = -45$

$$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(-45)}}{2(2)}$$

$$x = \frac{-14 \pm \sqrt{556}}{4}$$

$$x = \frac{-14 + \sqrt{556}}{4} \text{ and } x = \frac{-14 - \sqrt{556}}{4}$$

$x = 2.4$ and $x = -9.4$

But since x is a length it cannot be negative, so $x = 2.4$.

So, the breadth is 2.4 cm.

Question 15, (4)

Use SOHCAHTOA on triangle ABC to give

$$\sin A = \frac{8}{18}$$

$$A = sin^{-1}\left(\frac{8}{18}\right) = 26.4°$$

Area of triangle ADE is given by

$$\text{Area} = \frac{1}{2} de \sin A$$

$$= \frac{1}{2} d \sin 26.4$$

Since the area of ADE is 160 we have

$$\frac{1}{2} d(24) \sin 26.4 = 160$$

Multiply both sides by 2 to give

$$24 \sin 26.4\ d = 320$$

Rearranging gives

$$d = \frac{320}{24 \sin 26.4}$$

$$d = 30\ cm$$

So, $AE = 30\ cm$