Solutions

The Equation of a Tangent to a Curve

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Q1) a) $f(x) = 2x^2 + x - 3$ at $x = 2$

$f'(x) = 4x + 1$

$f'(2) = 9$

So, the gradient of the tangent at $x = 2$ is 9

b) $f(x) = -3x^2 + 7$ at $x = -2$

$f'(x) = -6x$

$f'(-2) = 12$

So, the gradient of the tangent at $x = -2$ is 12

c) $f(x) = x^2 + 4x - 4$ at $x = \frac{1}{2}$

$f'(x) = 2x + 4$

$f'\left(\frac{1}{2}\right) = 5$

So, the gradient of the tangent at $x = \frac{1}{2}$ is 5

d) $f(x) = x^3 + 5x - 6$ at $x = -1$

$f'(x) = 3x^2 + 5$

$f'(-1) = 8$

So, the gradient of the tangent at $x = -1$ is 8

Q2) a) $y = x^2 + 3x + 6$

Let $f(x) = x^2 + 3x + 6$

$f'(x) = 2x + 3$

$f'(1) = 5$

So, the gradient of the tangent at $x = 1$ is 5

Using $y - b = m(x - a)$ we have

$y - 10 = 5(x - 1)$

$y - 10 = 5x - 5$

$y = 5x + 5$

b) $y = x + \frac{1}{x} = x + x^{-1}$

Let $f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$f'(2) = \frac{3}{4}$

So, the gradient of the tangent at $x = 2$ is $\frac{3}{4}$

Using $y - b = m(x - a)$ we have

$y - \frac{5}{2} = \frac{3}{4}(x - 2)$

$y - \frac{5}{2} = \frac{3}{4}x - \frac{6}{4}$

$y = \frac{3}{4}x + 1$

c) $y = 4\sqrt{x} = 4x^{\frac{1}{2}}$

Let $f(x) = 4x^{\frac{1}{2}}$

$f'(x) = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$

$f'(9) = \frac{2}{3}$

So, the gradient of the tangent at $x = 9$ is $\frac{2}{3}$

Using $y - b = m(x - a)$ we have

$y - 12 = \frac{2}{3}(x - 9)$

$y - 12 = \frac{2}{3}x - 6$

$y = \frac{2}{3}x + 6$

d) $y = \cos x$ at $(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$

Let $f(x) = \cos x$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

So, the gradient of the tangent at $x = \frac{\pi}{6}$ is $-\frac{1}{2}$

Using $y - b = m(x - a)$ we have

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}(x - \frac{\pi}{6})$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}x + \frac{\pi}{12}$

$y = -\frac{1}{2}x + \frac{\pi}{12} + \frac{\sqrt{3}}{2}$