Solutions

Determining Local Extrema of a Function

Determining Local Extrema of a Function

Q1) a) $f(x) = 2x^2 + 3$

$f'(x) = 4x$

For SP's $f'(x) = 0$

$4x = 0$

$x = 0$

b) $g(x) = \frac{1}{2}x^3 - x + 2$

$g'(x) = \frac{3}{2}x^2 - 1$

For SP's $g'(x) = 0$

$\frac{3}{2}x^2 - 1 = 0$

$\frac{3}{2}x^2 = 1$

$x^2 = \frac{2}{3}, \; x = \pm\sqrt{\frac{2}{3}}$

c) $h(x) = x^3 - 2x^2 + x - 5$

$h'(x) = 3x^2 - 4x + 1$

For SP's $h'(x) = 0$

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}, x = 1$

d) $f(x) = x^3 - 3x^2 - 24x + 10$

$f'(x) = 3x^2 - 6x - 24$

For SP's $f'(x) = 0$

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = -2, x = 4$

Q2) a) $f(x) = 2x^4 - 12$

$f'(x) = 8x^3$

For SP's $f'(x) = 0$

$8x^3 = 0$

$x = 0$

When $x = 0, y = -12$

Stationary Point is at $(0, -12)$

b) $f(x) = 4x$

$f'(x) = 4$

Since the derivative is a constant there are no stationary points. This makes sense because the function, $4x$, is a straight line which has no stationary points.

c) $y = x + \frac{1}{x} = x + x^{-1}$

$\frac{dy}{dx} = 1 - x^{-2}$

For SP's $\frac{dy}{dx} = 0$

$1 - x^{-2} = 0$

$x^2 = 1$

$x = \pm 1$

When $x = 1, y = 1 + \frac{1}{1} = 2$

When $x = -1, y = -1 + \frac{1}{-1} = -2$

Stationary points are $(1, 2)$ and $(-1, -2)$

d) $y = x - 3\sqrt{x} = x - 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}} = 1 - \frac{3}{2\sqrt{x}}$

For SP's $\frac{dy}{dx} = 0$

$1 - \frac{3}{2\sqrt{x}} = 0$

$\frac{3}{2\sqrt{x}} = 1$

$\sqrt{x} = \frac{3}{2}, x = \frac{9}{4}$

When $x = \frac{9}{4}$

$y = \frac{9}{4} - 3\sqrt{\frac{9}{4}} = -\frac{9}{4}$

Stationary points is at $\left(\frac{9}{4}, -\frac{9}{4}\right)$

Q3) a) $y = \frac{1}{3}x^3 - 2x^2 + 3x - 1$

$\frac{dy}{dx} = x^2 - 4x + 3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, x = 3$

When $x = 1, y = \frac{1}{3} \cdot 1^3 - 2 \cdot 1^2 + 3 \cdot 1 - 1 = \frac{1}{3}$

When $x = 3, y = (\frac{1}{3} \cdot 3^3) - (2 \cdot 3^2) + (3 \cdot 3) - 1 = -1$

Stationary points are at $(1, \frac{1}{3})$ and $(3, -1)$

x	0	1	2	3	4
$\frac{dy}{dx}$	+	0	−	0	+
Shape	/	—	\	—	/

So, $(1, \frac{1}{3})$ is a maximum turning point and $(3, -1)$ is a minimum turning point.

b) $y = x^3 - 2x^2 - 4x + 1$

$\frac{dy}{dx} = 3x^2 - 4x - 4$

For stationary points $\frac{dy}{dx} = 0$

$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$x = -1.5, x = 2$

When $x = -1.5, y = 1.5^3 - (2 \cdot 1.5^2) - (4 \cdot 1.5) + 1 = -6.13$

When $x = 2, y = 2^3 - (2 \cdot 2^2) - (4 \cdot 2) + 1 = -7$

Stationary points are at $(-1.5, -6.13)$ and $(2, -7)$

x	-2	-1.5	1	2	3
$\frac{dy}{dx}$	$+$	0	$-$	0	$+$
Shape	/	—	\	—	/

So, $(-1.5, -6.13)$ is a maximum turning point and $(2, -7)$ is a minimum turning point.

c) $y = 2 + 5x - x^2 - x^3$

$\frac{dy}{dx} = 5 - 2x - 3x^2$

For stationary points $\frac{dy}{dx} = 0$

$5 - 2x - 3x^2 = 0$

$3x^2 + 2x - 5 = 0$

$(3x + 5)(x - 1) = 0$

$x = -\frac{5}{3}, x = 1$

When $x = -\frac{5}{3}, y = 2 + \left(5 \cdot -\frac{5}{3}\right) - \left(-\frac{5}{3}\right)^2 - \left(-\frac{5}{3}\right)^3 = 13.7$

When $x = 1, y = 2 + (5 \cdot 1) - 1^2 - 1^3 = 5$

Stationary points are at $(-\frac{5}{3}, 13.7)$ and $(1, 5)$

x	-2	$-\frac{5}{3}$	0	1	2
$\frac{dy}{dx}$	$-$	0	$+$	0	$-$
Shape	\	___	/	—	\

So, $(-\frac{5}{3}, 13.7)$ is a minimum turning point and $(1, 5)$ is a maximum turning point.

d) $y = x^3(x - 2)$

$y = x^4 - 2x^3$

$\frac{dy}{dx} = 4x^3 - 6x^2$

For stationary points $\frac{dy}{dx} = 0$

$4x^3 - 6x^2 = 0$

$2x^2(2x - 3) = 0$

$x = 0, x = 1.5$

When $x = 0, y = 0^4 - 2 \cdot 0^3 = 0$

When $x = 1.5, y = 1.5^4 - 2 \cdot 1.5^3 = -1.69$

Stationary points are at $(0, 0)$ and $(1.5, -1.69)$

x	-1	0	1	1.5	2
$\frac{dy}{dx}$	$-$	0	$-$	0	$+$
$Shape$	\	—	\	—	/

$(0, 0)$ is a falling point of inflection and $(1.5, -1.69)$ is a minimum T.P.

Q4) $y = x^4 - 32x^2$

$\frac{dy}{dx} = 4x^3 - 64x$

For stationary points $\frac{dy}{dx} = 0$

$4x^3 - 64x = 0$

$4x(x^2 - 16) = 0$

$4x(x + 4)(x - 4) = 0$

$x = -4, 0, 4$

$\frac{d^2y}{dx^2} = 12x^2 - 64$

When $x = -4, \frac{d^2y}{dx^2} = 128$

Since $\frac{d^2y}{dx^2} > 0$, the point is a minimum turning point

When $x = 0, \frac{d^2y}{dx^2} = -64$

Since $\frac{d^2y}{dx^2} < 0$, the point is a maximum turning point

When $x = 4, \frac{d^2y}{dx^2} = 128$

Since $\frac{d^2y}{dx^2} > 0$, the point is a minimum turning point