# Solutions

Determining Local Extrema of a Function

## Determining Local Extrema of a Function

Q1) a) $f(x) = x^3 - 2x^2 + x - 5, -2 \le x \le 1$

$f'(x) = 3x^2 - 4x + 1$

For stationary points $f'(x) = 0$, giving

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}, x = 1$

$f\left(\frac{1}{3}\right) = (\frac{1}{3})^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} - 5 = -4.85$

$f(1) = 1^3 - 2(1)^2 + 1 - 5 = -5$

$f(-2) = (-2)^3 - 2(-2)^2 - 2 - 5 = -23$

So, the greatest value is $-4.85$ and the least value is $-23$

b) $f(x) = x^3 - 3x^2 - 24x + 10, -1 \le x \le 6$

$f'(x) = 3x^2 - 6x - 24$

For stationary points $f'(x) = 0$, giving

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = -2, x = 4$

$f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 10 = 38$

$f(-1) = (-1)^3 - 3(-1)^2 - 24(-1) + 10 = 30$

$f(4) = 4^3 - 3(4^2) - 24(4) + 10 = -70$

$f(6) = 6^3 - 3(6^2) - 24(6) + 10 = -26$

So, the maximum value is 38 and the minimum value is $-70$

c) $g(x) = 6x^2 + 3x + 2, -5 \leq x \leq 5$

$g'(x) = 12x + 3$

For stationary points $g'(x) = 0$, giving

$12x + 3 = 0$

$x = -\frac{1}{4}$

$g\left(-\frac{1}{4}\right) = 6(-\frac{1}{4})^2 + 3\left(-\frac{1}{4}\right) + 2 = 1.625$

$g(-5) = 6(-5)^2 + 3(-5) + 2 = 137$

$g(5) = 6(5)^2 + 3(5) + 2 = 167$

So, the maximum value is 167 and the minimum value is 137

d) $f(x) = x^3(x - 2), -1 \leq x \leq 4$

$= x^3 - 2x^2$

$f'(x) = 3x^2 - 4x$

For stationary points $f'(x) = 0$, giving

$3x^2 - 4x = 0$

$x(3x - 4) = 0$

$x = 0, x = \frac{4}{3}$

Q2) a) $y = 4x^2 + 6x$

$\frac{dy}{dx} = 8x + 6$

For stationary points $\frac{dy}{dx} = 0$

$8x + 6 = 0$

$8x = -6, x = -\frac{3}{4}$

When $x = -\frac{3}{4}$, $\frac{d^2y}{dx^2} = 8$

Since $\frac{d^2y}{dx^2} > 0$, the point is a minimum turning point.

b) $y = x(x^2 - 4x - 3) = x^3 - 4x^2 - 3x$

$\frac{dy}{dx} = 3x^2 - 8x - 3$

For stationary points $\frac{dy}{dx} = 0$

$3x^2 - 8x - 3 = 0$

$(3x + 1)(x - 3) = 0$

$x = -\frac{1}{3}, \; x = 3$

$\frac{d^2y}{dx^2} = 6x - 8$

When $x = -\frac{1}{3}, \frac{d^2y}{dx^2} = -10$

Since $\frac{d^2y}{dx^2} < 0$, the point is a maximum turning point

When $x = 3, \frac{d^2y}{dx^2} = 10$

Since $\frac{d^2y}{dx^2} > 0$, the point is a minimum turning point

c) $y = x^4 - 12x^2$

$\frac{dy}{dx} = 4x^3 - 24x$

For stationary points $\frac{dy}{dx} = 0$

$4x^3 - 24x = 0$

$4x(x^2 - 6) = 0$

$x = 0, x = \pm\sqrt{6}$

$\frac{d^2y}{dx^2} = 12x^2 - 24$

When $x = 0, \frac{d^2y}{dx^2} = \ -24$

Since $\frac{d^2y}{dx^2} < 0$, the point is a maximum turning point

When $x = \pm\sqrt{6}, \frac{d^2y}{dx^2} = 12(6) - 24 = 48$

Since $\frac{d^2y}{dx^2} > 0$, these points are minimum turning points

d) $y = x^{\frac{1}{2}}(x - 6) = x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$

For stationary points $\frac{dy}{dx} = 0$

$\frac{3}{2}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} = 0$

$\frac{3}{2}x^{\frac{1}{2}} = \frac{3}{x^{\frac{1}{2}}}$

$\frac{3}{2}x = 3$

$3x = 6, x = 2$

$\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} + \frac{3}{2}^{-\frac{3}{2}}$

When $x = 2$

$\frac{d^2y}{dx^2} = \frac{3}{4\sqrt{2}} + \frac{3}{4\sqrt{2}}$

Since $\frac{d^2y}{dx^2} > 0$, the point is a minimum turning point